Quantitative Monographs
Correlation, De-correlation and Risk-Parity

Equities

Global
Quantitative

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How much does Risk-Parity owe to pairwise correlations?

The risk-parity portfolio construction technique aims to promote diversification, whilst limiting the exposure to estimation risk. We ask two important questions: (a) how do risk-parity allocations depend on the correlation structure of the investable universe and (b) do assets that de-correlate with the universe, and therefore bear a larger risk-parity weight, exhibit larger risk-adjusted returns in the cross-section?

Average pairwise correlation versus dispersion of pairwise correlations

Risk-parity allocation deviates from a simple inverse-volatility allocation, because it takes account of pairwise correlations. Contrary to expectations, it is not the average pairwise correlation (i.e. the level of co-movement) that is driving these deviations, but instead it is the way that pairwise correlations are dispersed around their mean. See Figure 1 for 90-day rolling estimates of these quantities for a universe of 35 futures contracts across all asset classes.

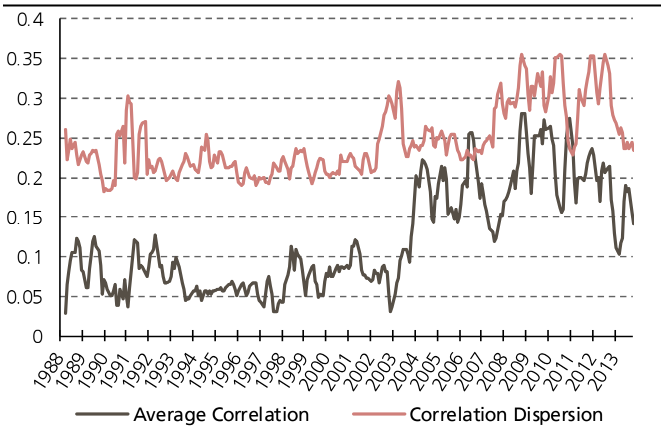
Extending low-risk patterns to a de-correlation anomaly across asset classes

Low risk/volatility patterns have long been identified within and across asset classes. Building on the outperformance of risk-parity strategies, we show that on a multi-asset-class setup, assets that de-correlate with the rest of the investable universe enjoy larger risk-adjusted returns above and beyond any low-volatility patterns. This result clearly benefits a risk-parity allocation, which typically over-weights assets that de-correlate.

Fixed income and the others

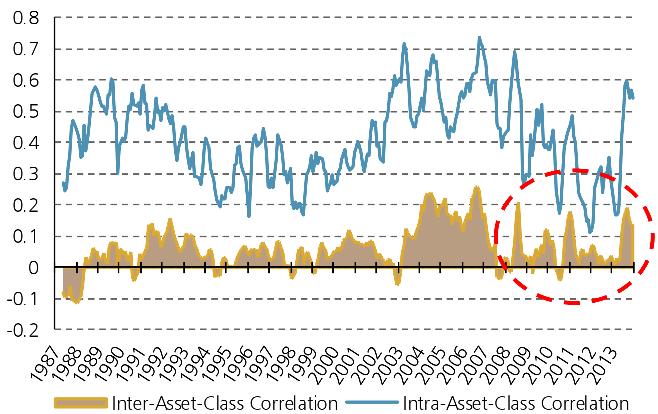
We confirm anecdotal evidence that fixed income assets have behaved relatively differently to all the other asset classes over and after the recent financial crisis. During this period, fixed income assets have exhibited very low correlation with the rest of the universe (see Figure 2) and this pattern has been claimed to be the main reason of the recent outperformance of risk-parity portfolios. However, we show that all our results are not just due to this fixed income decoupling, but instead remain robust even after excluding the fixed income assets entirely from the investable universe.

Figure 1: Average Correlation & Dispersion of Correlations



Source: UBS Quantitative Research. The figure presents the 90-day average pairwise correlation and dispersion of pairwise correlations across 35 futures contracts. Sample period: April 1988 December 2013.

Figure 2: Correlations of Fixed Income assets



Source: UBS Quantitative Research. The figure presents the 90-day average pairwise correlation across fixed income assets and between fixed income assets and all the non-fixed income assets. Sample period: April 1988 December 2013.

Introduction

One of the main research topics of our team over the last year has been the Risk-Parity portfolio construction methodology on which we have already published four research notes[1]. In the two most recent publications, "*Trend-Following meets Risk Parity*" (2 December 2013) and "*Risk-Parity versus Mean-Variance*" (16 May 2014), we even extended our focus of interest to a cross-asset-class allocation and highlighted the benefit from including the information from the correlation matrix in the weighting scheme of long-only and trend-following long-short portfolios. In our continuous interest on this topic and following client feedback on these publications, we now address some more general and fundamental questions that relate to the dependence of a risk-parity allocation and subsequently of portfolio performance on the pairwise correlations between assets and asset classes.

Entirely ignoring correlations might turn out worse than estimating them with error

There exists on-going discussion about whether correlations between assets can be efficiently and accurately estimated. For that reason, in calculating covariance matrices, researchers have suggested either entirely ignoring correlations or shrinking them towards some pre-specified level of long-term average that is often assumed to be equal to zero (see e.g. Ledoit and Wolf 2003, 2004a, 2004b). However, the evidence of increased co-movement during and after the recent financial crisis is unanimously accepted. The fact that correlations might indeed be hard to accurately estimate and unstable over time can be theoretically and empirically justified, but at other extreme, completely ignoring them in portfolio construction is even harder to justify, especially in this new era.

As already heavily highlighted in the literature (see Merton, 1980 and Chopra and Ziemba, 1993) and in our most recent publication on the topic, Markowitz's (1952) mean-variance portfolio methodology, even though it does properly account for asset correlations, it is largely affected by estimation error. As an alternative to these empirical difficulties of mean-variance, the risk-parity portfolio construction technique has been recently introduced (Qian, 2005 and Maillard, Roncalli and Teiletche, 2010) as a way to account for the correlation structure of the investable universe and therefore promote diversification, whilst limiting the exposure to estimation risk. The outperformance of risk-parity allocations over the last few decades has received significant publicity and our aim is to comprehend the idiosyncratic features of the methodology in terms of allocating weights and therefore to explore its performance drivers.

Research questions & findings

We decide to structure our analysis in two relatively related but still distinct pillars that aim to elaborate on two broad research questions:

1. *How do risk-parity allocations depend on the correlation structure of the investable universe?*

   The first section focuses explicitly on the nature of risk-parity allocations and their dependence on the overall correlation structure of the investable universe, i.e. on the cross-sectional distribution of all pairwise correlations of portfolio constituents. Contrary to expectations, we find that it is the second

[1] See the list of our recent UBS publications on Risk-Parity:

- *Understanding Risk Parity* (7 February 2013)
- *Risk Parity with different Risk Measures* (10 July 2013)
- *Trend-Following meets Risk Parity* (2 December 2013)
- *Risk-Parity versus Mean-Variance* (16 May 2014)

moment (dispersion of correlations) and not the first moment (average correlation) that is driving the risk-parity allocations.

2. *Do assets that de-correlate with the universe, and therefore bear a larger risk-parity weight, exhibit larger risk-adjusted returns in the cross-section?*

   Motivated by the recent outperformance of risk-parity strategies, the second pillar asks this even more general question. We find that on a multi-asset-class setup, assets that de-correlate with the rest of the investable universe do enjoy larger risk-adjusted returns above and beyond the well-documented low-volatility patterns. This result clearly benefits a risk-parity allocation, even though it seems to be conflicting with traditional asset pricing theories (Cochrane, 2000). As explained later on, we call this the "*de-correlation anomaly*". Overall, it can be claimed that this pattern is a critical performance driver of risk-parity allocations.

Data Description

35 futures contracts across all asset classes

The empirical analyses of this research paper use exactly the same dataset that we used in our most recent publication on the risk-parity front, the Quant Keys "*Risk-Parity versus Mean-Variance*" (16 May 2014). In particular, our universe includes 35 futures contracts (6 energy contracts, 10 commodity contracts, 6 fixed income contracts, 6 foreign exchange contracts and 7 equity index contracts – see Appendix A for details) between January 1987 and December 2013. We obtain daily prices of the generic ratio backwards-adjusted continuous price-series from Bloomberg and calculate fully-collateralised monthly returns in *excess* of the prevailing risk-free rate for each futures contract. For further details on the data handling procedure please see the Global Quantitative Research Monograph "*Trend-Following meets Risk Parity*" (2 December 2013) and Appendix A therein.

Who should read this report?

This report addresses broad questions and offers insight across a number of different dimensions. Risk managers will find it interesting because it explores the correlation dynamics and their impact on portfolio construction and performance. Portfolio managers will find the empirical evidence that de-correlation is compensated in the cross-section rather compelling, as it contradicts with traditional asset pricing theories, which claim that the priced component of asset returns is the one that co-varies with the market (Cochrane, 2000). Moreover, our extension of low-volatility patterns to more general de-correlation patterns across asset classes will definitely attract the attention of asset managers.

The rest of the note contains a brief theoretical section that outlines the testable hypotheses that are addressed and then continues with the two main sections of our analysis.

Theoretical Motivation

We start this brief theoretical section by first describing one of the simplest risk-based portfolio construction methodologies[2], the volatility-parity (VP) scheme[3]. This scheme allocates weights that are inversely proportional to the asset volatilities:

$$w_i^{VP} \propto \frac{1}{\sigma_i} \tag{1}$$

The reason why this weighting scheme is called volatility-parity is because all assets enter the portfolio with the same ex-ante volatility, hence the parity. Assets with lower (higher) volatilities cross-sectionally would then naturally have larger (smaller) relative weights in the portfolio. Trivially, when all assets have the same volatility, VP results in an equally-weighted portfolio.

More interestingly, if all pairwise correlations between the assets are equal, it can be shown that VP splits the portfolio volatility equally across the portfolio constituents. However, equality between pairwise correlations is a very simplistic assumption to make in practice. In order to achieve equal proportional contribution to portfolio volatility from all the assets, one has to resort to a more sophisticated weighting scheme, known as risk-parity (RP). In other words, RP solves for a weight allocation that satisfies the following condition:

$$w_i^{RP} \cdot MCR_i = constant, \forall i \tag{2}$$

where MCR is the *marginal contribution to risk* of asset i, i.e. the change in portfolio volatility for a marginal change in the weight of asset i. According to this objective, the *weighted* marginal contribution to risk (also known as *total contribution to risk* or *percentage contribution to risk* in the literature) becomes equal (across all assets) to a constant that is simply $\frac{1}{N}$th of portfolio volatility $\sigma_P(\boldsymbol{w})$.

It can be shown (see Appendix B) that the MCR of an asset is equal to the product between its volatility σ_i and its correlation with the overall portfolio, $\rho_{i,P}(\boldsymbol{w})$:

$$MCR_i = \frac{\partial \sigma_P(\boldsymbol{w})}{\partial w_i} = \sigma_i \cdot \rho_{i,P}(\boldsymbol{w}) \tag{3}$$

Combining equations (2) and (3) leads to a very important result:

$$w_i^{RP} \propto \frac{1}{MCR_i} = \frac{1}{\sigma_i} \cdot \frac{1}{\rho_{i,P}(\boldsymbol{w}^{RP})} \tag{4}$$

Contrary to the VP allocation, RP assigns larger weights not only to assets with lower volatilities in the cross-section, but also to assets that correlate least with the overall portfolio. It is critical to notice that the result of equation (4) is *not* a closed-form expression for the RP weighting scheme, because the correlation $\rho_{i,P}(\boldsymbol{w}^{RP})$ at the right hand side is a function of all asset weights and is therefore endogenous,

[2] For an overview of risk-based allocation schemes, see for instance Lee (2011) and Leote de Carvalho, Lu and Moulin (2012) as well as references therein.
[3] Various other terms have been used for a volatility-parity allocation: inverse-volatility scheme, volatility-timing, and volatility-scaling.

hence the explicit dependence on the weight vector $\boldsymbol{w}^{RP}$ in the notation. However, this expression can nicely illustrate the relationship between RP and VP solutions.

Combining equation (1) and (4) yields:

$$\frac{w_i^{RP}}{w_i^{VP}} \propto \frac{1}{\rho_{i,P}(\boldsymbol{w}^{RP})} \tag{5}$$

Notice that this result is a proportionality statement and therefore it should only result in qualitative conclusions. Along these lines:

- When the correlation of an asset with the rest of the portfolio drops (we can introduce the term "*de-correlation*" here), then its RP weight increases *relative* to its inverse-volatility weight.
- When an asset correlates more with the rest of the portfolio, then its RP weight decreases *relative* to its inverse-volatility weight.

In order to make absolute statements on the value of the weights (which is larger, which is smaller), one has to determine the factors of proportionality in (1) and (4); these are trivially defined so that the weights add up to 100% ("fully-invested"). See Appendix C for further details.

Testable Hypotheses

The theoretical analysis above can give rise to two streams of testable hypotheses, which constitute the two main pillars of focus of this research note. These are outlined below:

A. Correlation Dynamics & Diversification (starts in page 6):

Following from equation (5), VP and RP weighting schemes differ as a function of the correlation dynamics of the universe. The RP allocation aims to make proper use of pairwise correlations in order to increase portfolio diversification, but

- What are really the (correlation) states of the market that the two allocations significantly differ?
- How is portfolio performance affected by the different allocations in periods when these allocations do significantly differ?

B. A De-correlation Anomaly (starts in page 16):

Following from equation (4), RP over-weights the low-volatility assets and the assets that de-correlate with the rest of the universe in an effort to increase portfolio diversification. From an investment point of view, it is worth exploring whether the assets that are over-weighted by RP also tend to exhibit larger risk-adjusted returns in the cross-section. In particular:

- Do low-volatility assets exhibit larger risk-adjusted returns in the cross-section?
- Do de-correlated assets exhibit larger risk-adjusted returns in the cross-section?

Low-volatility and low-beta patterns have long been documented in equity markets (e.g. Black, Jensen and Scholes, 1972, Ang, Hodrick, Xing and Zhang 2006, 2009), but very little evidence exists for a multi-asset-class universe; the only exception is the very recent paper by Frazzini and Pedersen (2014).

The two main sections of this research paper empirically test the above hypotheses.

Correlation Dynamics & Diversification

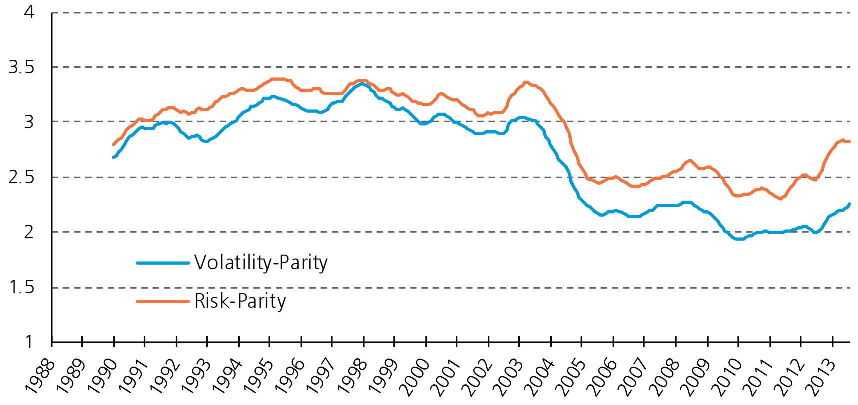
Choueifaty and Coignard (2008) introduce the diversification ratio (DR), which is the ratio of the weighted sum of asset volatilities divided by the portfolio volatility:

$$DR(\boldsymbol{w}) = \frac{\boldsymbol{w}^T \cdot \sqrt{diag(\boldsymbol{\Sigma})}}{\sqrt{\boldsymbol{w}^T \cdot \boldsymbol{\Sigma} \cdot \boldsymbol{w}}} = \frac{\sum_{i=1}^{N_t} \sigma_i \cdot w_i}{\sigma_P(\boldsymbol{w})} \tag{6}$$

Assuming a long-only portfolio, when all pairwise correlations are equal to 1, the diversification ratio takes its minimum value, that of 1. Clearly, different weighting schemes would result in different values of the ratio, with larger values implying greater amount of induced diversification due to the weighting scheme. Being out of the scope of this research note, the weighting scheme that maximises the DR, is termed the *most-diversified portfolio* by Choueifaty and Coignard (2008).

In order to evaluate the composition of the VP and RP long-only portfolios in terms of promoting diversification, we plot (the 24-month moving average of) their respective *realised* diversification ratios in Figure 3. A realised rolling calculation of equation (6) means that we use weights estimated as of time $t-1$ and realised measures of volatilities as of t. This is because in a real-life scenario, weights are first determined, a portfolio is then constructed and held for a month before any weight rebalancing takes place.

Figure 3: Realised Diversification Ratio of VP and RP long-only portfolios



Source: UBS Quantitative Research. The figure presents the time evolution of a 24-month moving average of the realised diversification ratio of a volatility-parity long-only strategy and a risk-parity long-only strategy. The sample period is from April 1988 to December 2013.

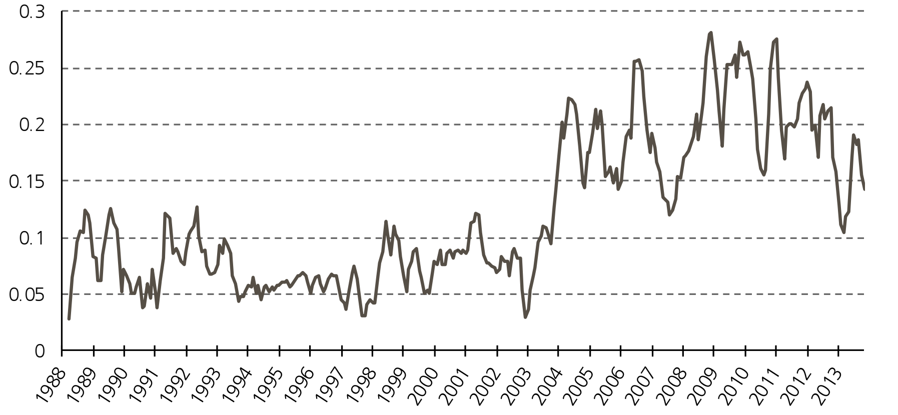
Two important key take-aways:

- The risk-parity allocation increases the diversification of the portfolio compared to a simple inverse-volatility weighting scheme.
- The level of absolute diversification has fallen since 2004, but at the same time the relative diversification benefit from risk-parity has increased.

Both of these findings can be related to the level of pairwise correlations. In a universe of assets that are mildly correlated, overall diversification is larger (using more technical term, a larger number of principal components is necessary to explain the variation of the universe). When asset co-movement becomes more pronounced, then the level of potential diversification falls, but at these states of the market a risk-parity allocation that accounts for the correlation effects can generate a larger relative benefit when compared to a simple scheme that ignores completely the off-diagonal elements of the correlation matrix.

The above hypothesis can be empirically tested. To start with, we present in Figure 4 the time evolution of a 90-day estimate of pairwise correlation of our universe.

Figure 4: Average Pairwise Correlation

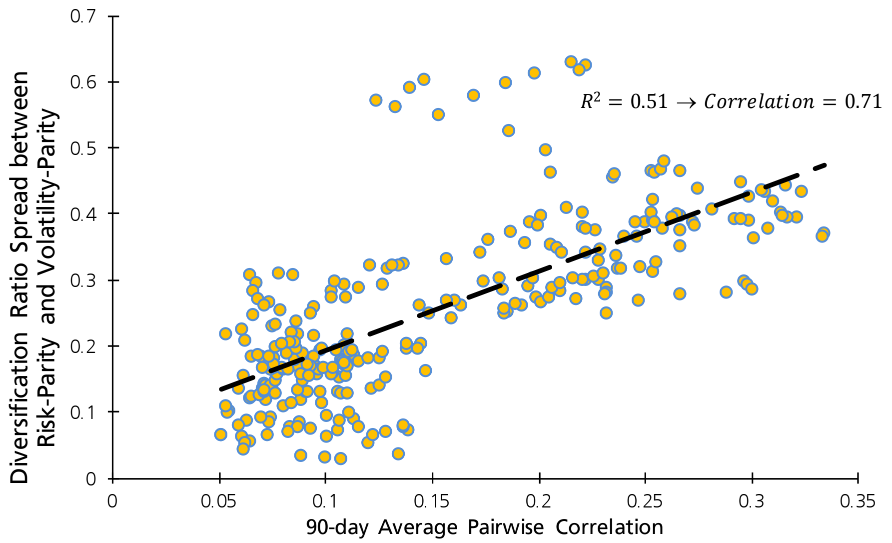


Source: UBS Quantitative Research. The figure presents a 90-day measure of average pairwise correlation across all futures contracts. The sample period is from April 1988 to December 2013.

The level of co-movement has dramatically increased since 2004

As already identified in the literature, the level of market co-movement has dramatically increased over the last decade and this is clearly one of the reasons why the absolute level of diversification has fallen in Figure 3. Our objective is to uncover how these patterns have affected VP and RP allocations. Figure 5 presents a scatterplot between the above measure of average pairwise correlation and the spread between the diversification ratios of VP and RP allocations in Figure 3. It is obvious that the benefit in the induced diversification by the more sophisticated weighting scheme of risk-parity becomes more pronounced in periods of larger average pairwise correlation when diversification benefits seem to vanish.

Figure 5: Average Pairwise Correlation versus Diversification Ratio Spread

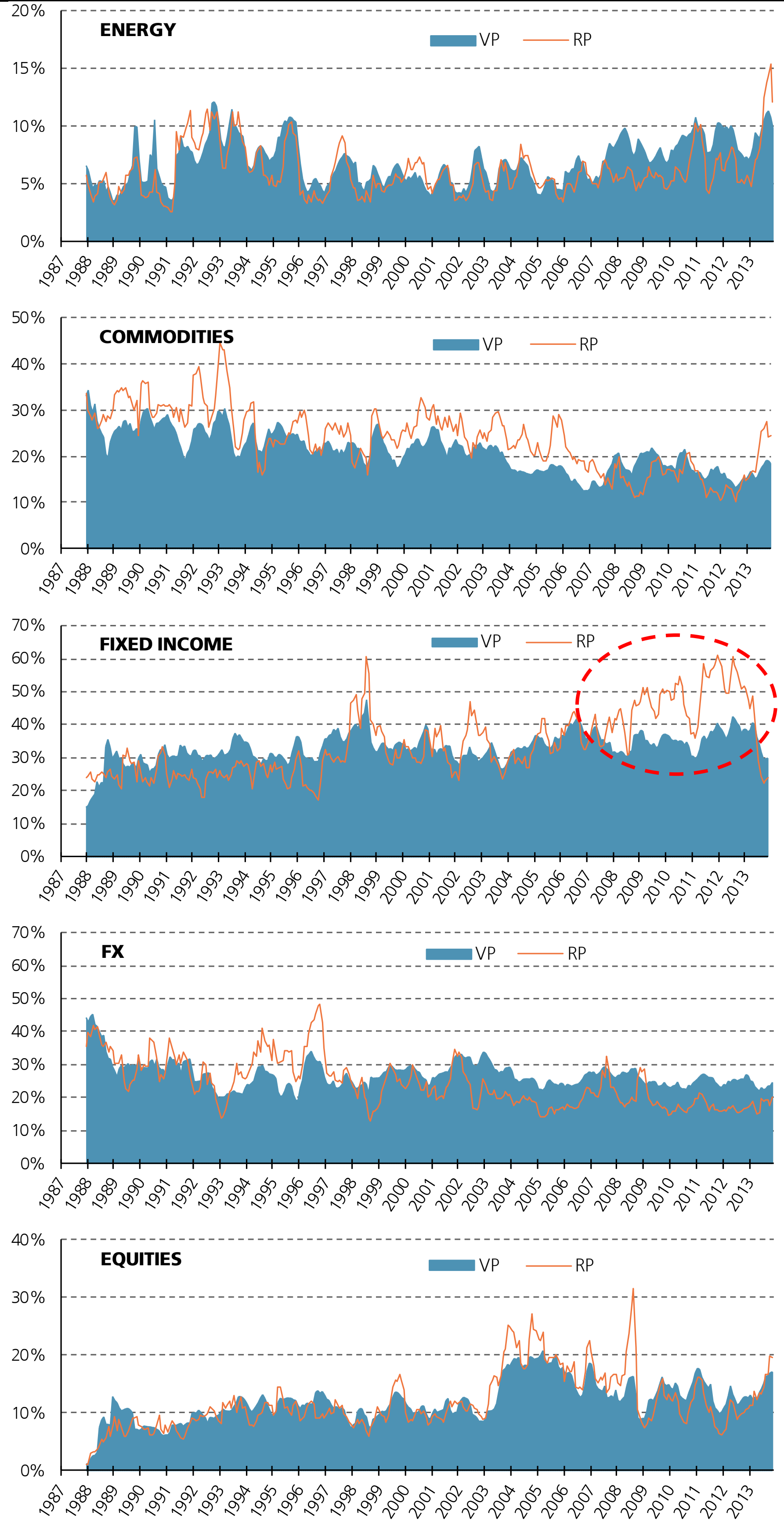


Source: UBS Quantitative Research. The figure presents a scatterplot between a 90-day measure of average pairwise correlation across all futures contracts against the spread between the diversification ratio of a volatility-parity allocation and a risk-parity allocation. The sample period is from April 1988 to December 2013.

How do VP and RP portfolio allocations differ?

Following this finding, we decide to take a closer and more careful look into the way that VP and RP allocations differ over time so that at a subsequent step relate the intertemporal variations of their mutual deviations to the prevailing correlation conditions. Figure 6 presents the time evolution of VP and RP allocations grouped by asset class for standard long-only strategies across all futures contracts.

Figure 6: Volatility-Parity versus Risk-Parity Long-Only Weights per Asset Class



Source: UBS Quantitative Research. Volatility-parity (VP) and risk-parity (RP) weights per asset class.

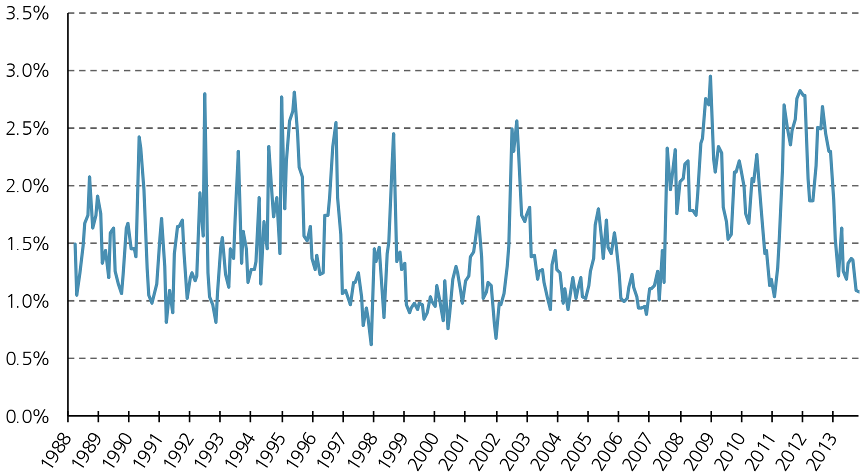
As one would expect, the two weighting schemes follow similar patterns, but there are periods when the deviations among each other are rather pronounced like the highlighted 2008-2012 period for fixed income contracts. Clearly, pairwise correlations come in to play.

In order to quantify the deviations between VP and RP weighting schemes we calculate at the end of each month the root mean square deviation (Euclidean distance) between the two weight vectors:[4]

$$RMSD_t = \sqrt{\frac{\sum_{i=1}^{N_t}\left(w_{VP,t}^i - w_{RP,t}^i\right)^2}{N_t}} \tag{7}$$

where N_t is the number of available contracts at the end of month t. Figure 7 presents the time-evolution of the deviation measure (as a reminder, the weighting schemes are estimated using a window of the past 90 trading days).

Figure 7: 90-day Root Mean Square Deviation between VP and RP weights



Source: UBS Quantitative Research. The figure presents a 90-day measure of the root mean square deviation between volatility-parity and risk-parity weights. The sample period is from April 1988 to December 2013.

The units in the plot are weight percentages, so the VP and RP weights of each asset deviate on average by about 1.5% across the entire sample period.

As the $RMSD$ is non-zero and more importantly time-varying, it's worth investigating whether its variation can be explained by the contemporaneous levels of pairwise correlation (as per Figure 4). For that reason, we regress the monthly levels of $RMSD$ on the contemporaneous levels of average pairwise correlation $\bar{\rho}$ and test the statistical significance of the slope coefficient of the latter. If that is positive and statistically significant then the RP weights deviate from VP weights in periods of high average pairwise correlation. The result of the regression is:

$$\underset{}{RMSD_t} = \underset{(5.97)}{0.01} + \underset{(1.10)}{0.02} \cdot \bar{\rho}_t + \epsilon_t \qquad (R^2 = 8.07\%) \tag{8}$$

where the t-statistics are calculated using Newey and West (1987) robust standard errors with 4 monthly lags (because both $RMSD_t$ and $\bar{\rho}_t$ are estimated using a 90-day estimation window).

[4] All the results of this section remain unchanged if instead of the root mean square deviation, we use the mean absolute deviation, defined as: $MAD_t = \frac{1}{N_t}\sum_{i=1}^{N_t}\left|w_{VP,t}^i - w_{RP,t}^i\right|$.

Even though the coefficient of the average pairwise correlation is positive, it is not statistically significant, so it is not strictly accurate to claim that a risk-parity allocation scheme deviates significantly from a simple inverse-volatility weighting scheme in states of high correlation. It appears that this was just the first stop of our journey in understanding how RP allocations differ from VP allocations.

It is not (just) the average pairwise correlation that matters

So far we have made a – what it was proven to be – slightly uneducated guess: "*when the average pairwise correlation changes, RP allocation differs from VP allocation*". The empirical evidence did not significantly support it, so we take a further step and think more carefully about the relationship between the two weighting schemes and the way that they differ over time.

Clearly, the two schemes are identical when all pairwise correlations are the same. What we next intend to investigate is not only how the change in $\bar{\rho}$ determines the deviation between the two schemes, but more importantly *how the way by which* $\bar{\rho}$ changes determines this deviation.

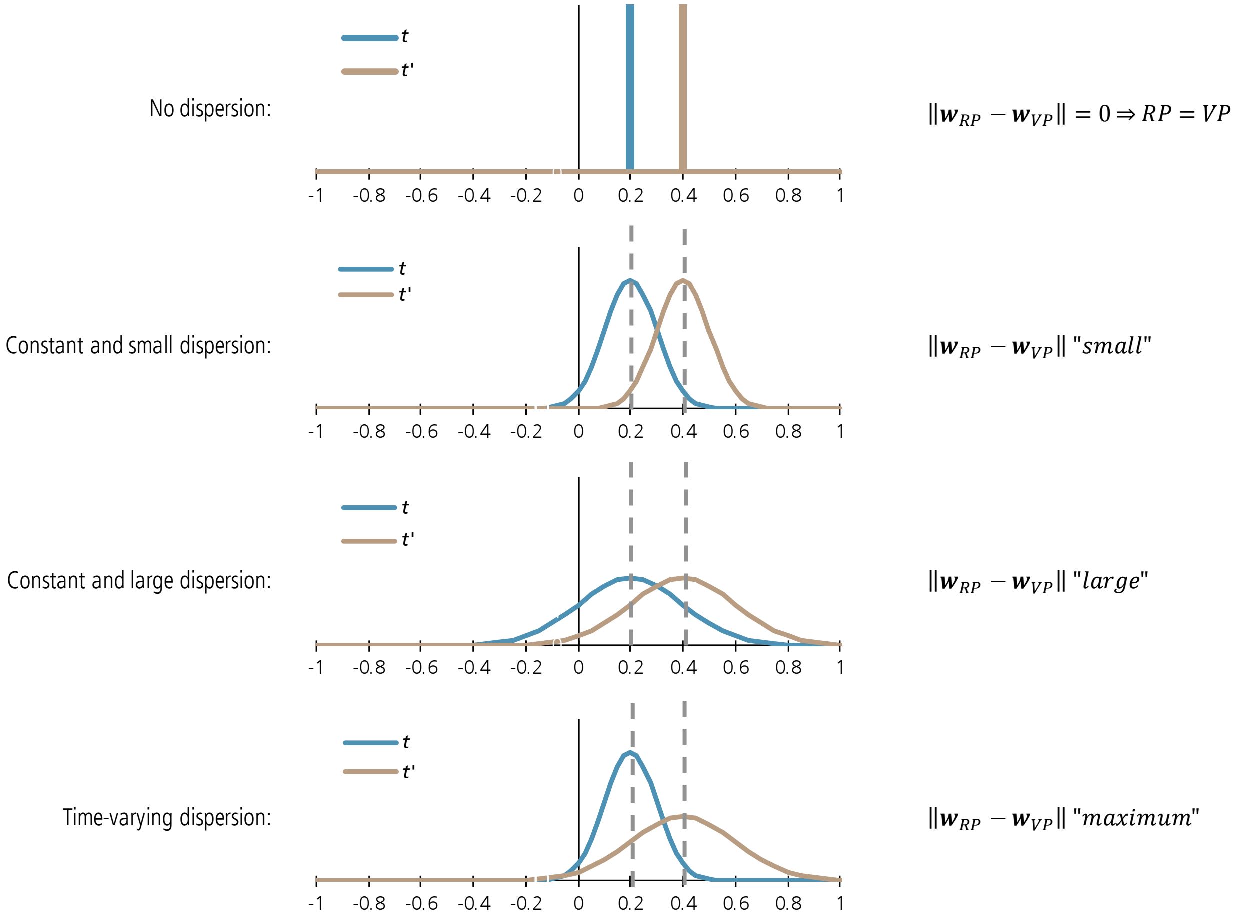
For the sake of illustration, suppose that at some time t, all pairwise correlations are the same and equal to 0.2; trivially, $\overline{\rho_t} = 0.2$. Now consider a time $t' > t$ when the average pairwise correlation increases to $\overline{\rho_{t'}} = 0.4$. There is an infinite number of scenarios that can lead to an increase of the average pairwise correlation by 0.2:

- If all pairwise correlations increase uniformly by 0.2.
- Or if half of the pairwise correlations increase to 0.6, and the rest remain unchanged to 0.2.
- Or if half of the pairwise correlations increase to 0.7, and the other half drop to 0.1.
- Or …

In all the above scenarios, $\overline{\rho_{t'}} = 0.4$; however, the *distribution* of pairwise correlations at time t' is very different from one scenario to another. It is only in the first scenario that the transition between t and t' would generate exactly the same allocations (and therefore rebalancing) between RP and VP schemes. All other scenarios would result in different allocations between the two schemes and our intuition is that when the pairwise correlations are more ***dispersed*** around their mean the deviations should increase. To illustrate this further, see Figure 8, which presents different scenarios of dispersion (standard deviation) of pairwise correlations. Notice that the distributions do not have to be normal – and are not in practice – but they are only presented as such for the sake of illustration.

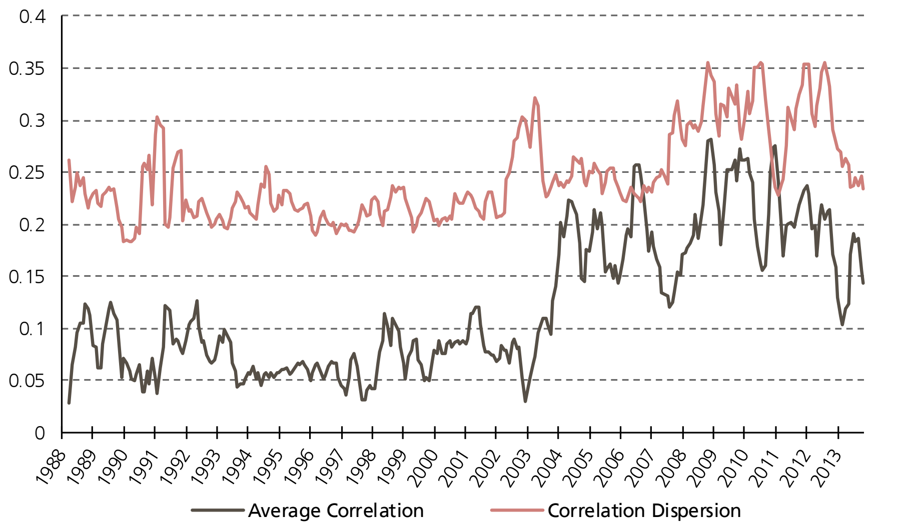
Following the above thinking process, it seems worth inspecting how the dispersion of pairwise correlations has evolved over time. Figure 9 presents the time-series of 90-day estimates of the dispersion of pairwise correlations across all futures contracts. The figure includes the average pairwise correlation as well (already presented in Figure 4) for visual inspection of the patterns. Not only is the dispersion of pairwise correlations non-zero, but more importantly it is time-varying. Furthermore, it does not appear to always pick at times when the average pairwise correlation picks. There are periods of significant co-movement with low dispersion (e.g. 2004-2007) as well as periods of very low average correlation, but with large dispersion (e.g. 2002-2004), which implies existence of large positive and negative correlations among pairs that are however distributed evenly around zero.

Figure 8: Distributions of Pairwise Correlations – Case Study



Source: UBS Quantitative Research. The figure presents a number of scenarios of the distribution of pairwise correlations between two time instances t and $t' > t$.

Figure 9: Average Pairwise Correlation and Dispersion of Correlations



Source: UBS Quantitative Research. The figure presents a 90-day measure of the average pairwise correlation and the dispersion of pairwise correlations across all futures contracts. The sample period is from April 1988 to December 2013.

For a more detailed inspection on the way that the cross-sectional distribution of pairwise correlations has changed over time see Appendix D.

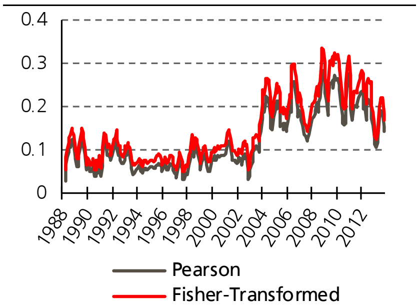
A technical point on the estimation of correlation

A technical, but still very important point that has to be stressed at this stage (and should not be hidden in an appendix): as a mathematical operation, the ordinary correlation coefficient (also known as the Pearson correlation) is bounded in the $[-1,1]$ region of values. As a consequence, when the average pairwise correlation approaches the upper bound of 1, the distribution of correlations becomes progressively more asymmetric, because any pairwise correlation that lies above the mean cannot numerically take values greater than 1. As a consequence, the deviation of these values from the mean becomes small and overall the dispersion of pairwise correlations might appear to shrink as the average pairwise correlation reaches extreme values. This artificial negative relationship between the mean and the standard deviation of the distribution of pairwise correlations can potentially swamp the econometric analysis (because of multicollinearity) and should be carefully dealt with.

The most popular remedy to this problem is to estimate the pairwise correlations using the Fisher (1915) transformation, which maps the region of values $[-1,1]$ into $[-\infty,\infty]$ and generates approximately normally distributed correlation estimates under certain conditions.[5,6] With the Fisher-transformed correlation estimates, we can then calculate the average and the standard deviation of the distribution, without having any artificially induced correlation between the two.

All averages and dispersion estimates of pairwise correlations that are used for any econometric analysis in this research note are based on Fisher-transformed estimates. Given that the average pairwise correlation across all asset classes (see Figure 9) is not reaching extreme values, the difference between Fisher-transformed and raw Pearson correlation estimates is minuscule (see Figure 10 for a visual comparison of the average correlation; the picture for the correlation dispersion is very similar). However, this transformation can become critical if a similar analysis conducted within a single asset class that correlations tend to be relatively larger.

Figure 10: Fisher Transformation



Source: UBS Quantitative Research. The figure presents a 90-day measure of the average pairwise correlation using either simple (Pearson) correlation estimates or Fisher-transformed correlation estimates. The sample period is from April 1988 to December 2013.

Regression Analysis

Returning to our empirical investigation of the difference between RP and VP allocations and given the fact that, empirically, the correlation dispersion is non-zero and time-varying, we extend the regression analysis of equation (8) and regress the monthly levels of $RMSD$ on the first two moments of the distribution of pairwise (Fisher-transformed) correlations: the average $\bar{\rho}$ and the standard deviation (dispersion) $\sigma(\bar{\rho})$:

$$RMSD_t = const. + b \cdot \overline{\rho}_t + c \cdot \sigma_t(\rho_t) + \epsilon_t \tag{9}$$

The results are presented in Figure 11 (the results from equation (8) are also included for comparison purposes).

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[5] The conditions are (a) that the two variables X and Y, for which we calculate the correlation, follow a bivariate normal distribution and (b) that any pair of values (X_i, Y_i) is independent from any other pair of values of the two vectors X and Y.

[6] The Fisher transformation is simply the following operator applied to an estimate ρ of the Pearson correlation coefficient between two variables:

$$z = \frac{1}{2}\ln\left(\frac{1+\rho}{1-\rho}\right)$$

This transformation is the standard textbook approach that is used in order to calculate confidence intervals for correlation estimates.

Figure 11: Regression Results for the Root Mean Squared Deviation

Dependent Variable: $RMSD$			
const.	0.01	0.00	0.00
	(5.97)	(-0.30)	(-0.62)
$\bar{\rho}$ [Fisher]	0.02		-0.01
	(1.10)		(-0.46)
$\sigma(\rho)$ [Fisher]		0.05	0.06
		(3.37)	**(3.86)**
Adjusted R^2(%)	8.07	22.41	22.60

Source: UBS Quantitative Research. The table presents the regression results from regressing the 90-day measure of the root mean square deviation ($RMSD$) between volatility-parity and risk-parity weights on the two moments of the distribution of pairwise (Fisher-transformed) correlations: the average $\bar{\rho}$ and the standard deviation (dispersion) $\sigma(\bar{\rho})$. The t-statistics are calculated using Newey and West (1987) robust standard errors with 4 lags. The sample period is from April 1988 to December 2013.

The dispersion of pairwise correlations controls the relationship between VP and RP allocations

The findings are insightful. Contrary to our initial (and intuitive) claim that the level of pairwise correlations should control the deviations of risk-parity weights from $1/\sigma$, it is, in fact, solely the dispersion of pairwise correlations that significantly (at the strictest level of 1%) leads to any significant deviations between the two weighting schemes. Moreover, the interpretation of the sign of the slope coefficient is that the difference in the two weighting schemes is more pronounced in high dispersion of correlations states of the market.

Is this result due to the fixed income assets?

Following from Figure 6, one of the largest wedges between VP and RP weights has been the 2008-2012 period for fixed income contracts. As we mentioned in the cover page of this research note and we will also elaborate further in the next section, the fixed income asset class has generally behaved very differently to the rest of the asset classes for the period over and after the recent financial crisis. It is therefore very natural to ask whether our results hold if we were to exclude the fixed income assets from our universe.

For the sake of completeness, we repeat the regression analysis of Figure 11 after excluding one asset class at a time and present the results in Figure 12 below.

Figure 12: Regression Results after excluding one asset class at a time

	ALL	ex. NRG	ex. CMDTY	ex. FI	ex. FX	ex. EQ
const.	0.00	-0.02	-0.03	0.00	-0.02	-0.03
	(-0.62)	**(-3.74)**	**(-4.55)**	(0.58)	**(-2.85)**	**(-2.86)**
$\bar{\rho}$ [Fisher]	-0.01	0.00	-0.01	-0.01	0.01	-0.01
	(-0.46)	(0.02)	(-0.50)	**(-1.76)**	(0.55)	(-0.96)
$\sigma(\rho)$ [Fisher]	0.06	0.10	0.10	0.04	0.09	0.12
	(3.86)	**(5.01)**	**(5.78)**	**(2.13)**	**(4.14)**	**(5.02)**
Adj. R^2(%)	22.60	54.22	55.40	14.34	53.81	37.91

Source: UBS Quantitative Research. The table replicates the multi-variable regression of Figure 11 after excluding one asset class at a time (the first column simply repeats the result using all asset classes). The t-statistics are calculated using Newey and West (1987) robust standard errors with 4 lags. The sample period is from April 1988 to December 2013.

Our result remains unchanged no matter which asset class we exclude from the universe

Even though the statistical significance slightly falls from 1% level to 5% level when the fixed income assets are excluded from the universe, our main result remains unchanged. The deviation between VP and RP weights is largely determined by the way that pairwise correlations are dispersed around their average value and not (just) by the average value itself. Needless to say that exclusion of any other asset class makes our finding statistically stronger.

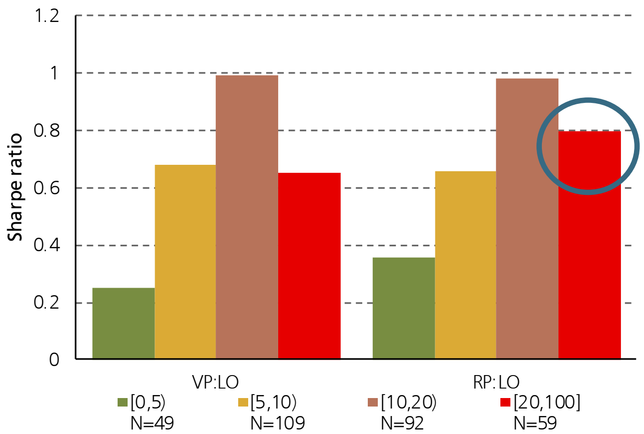
Performance and Dispersion of Pairwise Correlations

Correlation event study

In our recent Global Quantitative Research Monograph "*Trend-Following meets Risk Parity*" (2 December 2013), we presented a correlation event study, by splitting the months of the sample period based on the level of average pairwise correlation and then calculating the risk-adjusted performance of long-only and more importantly trend-following strategies across the various correlation regimes. The outcome of the analysis is that the performance of the strategies is largely dependent on the level of pairwise correlation of the constituent assets, with large levels of co-movement being associated to drawdowns and poor performance. Instead, a risk-parity allocation has been shown to significantly reduce the drawdowns is such states of the market.

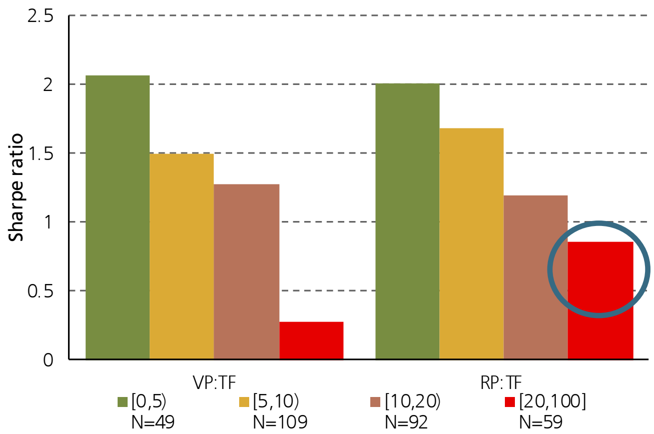
Using the revised universe of this research note, we replicate the correlation event-study and present it separately for long-only strategies in Figure 13 and for trend-following strategies in Figure 14.[7] Our main focus is the high-correlation regime (red bars). Clearly, if there is any difference in the performance of the strategies between a volatility-parity allocation and a risk-parity allocation, then this is largely pronounced in this market regime. When co-movement is extreme, this is exactly when a more robust portfolio construction technique can increase diversification and safeguard against drawdowns.

Figure 13: Correlation Event Study – Long-Only



Source: UBS Quantitative Research. The figure presents the annualised Sharpe ratio of a volatility-parity long-only strategy ($VP{:}\,LO$) and a risk-parity long-only strategy ($RP{:}\,LO$) for four different states of average pairwise correlation: between 0% and 5%, 5% and 10%, 10% and 20% and above 20%. The number of months N for each correlation bucket is shown in the legend. The sample period is from April 1988 to December 2013.

Figure 14: Correlation Event Study – Trend-Following



Source: UBS Quantitative Research. The figure presents the annualised Sharpe ratio of a volatility-parity trend-following strategy ($VP{:}\,TF$) and a risk-parity trend-following strategy ($RP{:}\,TF$) for four different states of average pairwise correlation: between 0% and 5%, 5% and 10%, 10% and 20% and above 20%. The number of months N for each correlation bucket is shown in the legend. The sample period is from April 1988 to December 2013.

Following the results of this research note, it is worth conducting a more granular analysis and break down the performance benefit of a risk-parity allocation into correlation-dispersion regimes.

Correlation-dispersion event study

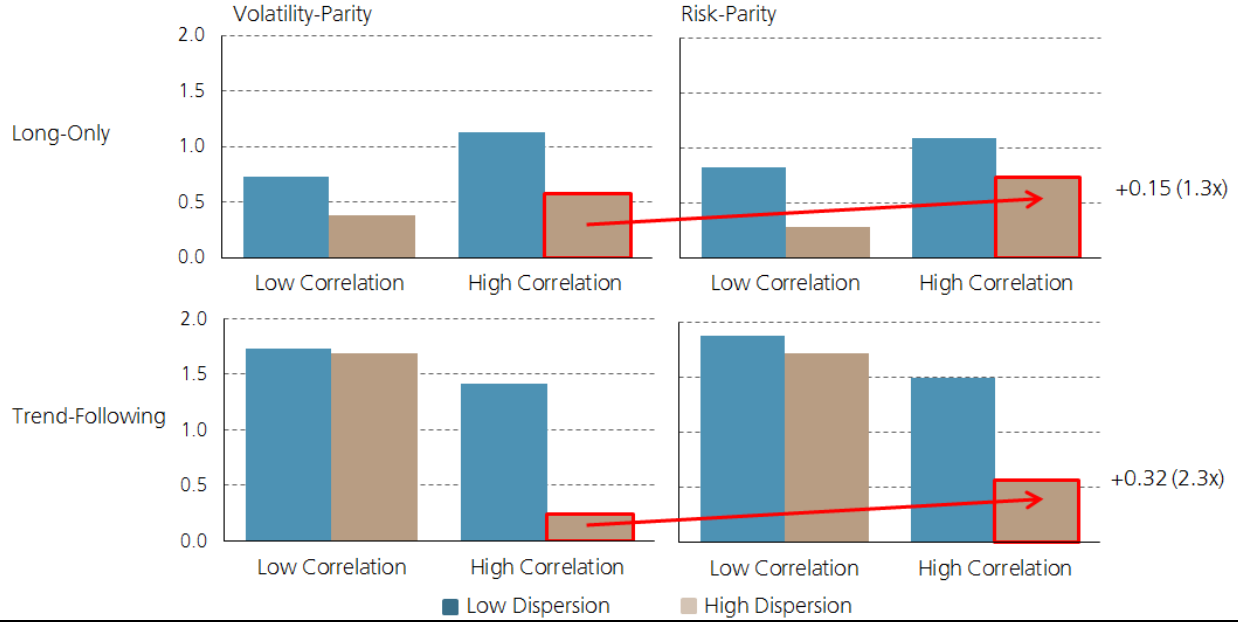
For that reason, we split the 309 months of our data sample (April 1988 – December 2013) based on the level of average pairwise correlation into low and high correlation months. As a second step, we split these two correlation buckets separately into low and high dispersion buckets so that we can form four

[7] All strategies for this particular analysis are constructed with a 10% volatility target. For further information on the construction of the strategies, refer to our Global Quantitative Research Monograph "*Trend-Following meets Risk-Parity*" (2 December 2013) and to our recent Quant Keys "*Risk-Parity versus Mean-Variance*" (16 May 2014).

correlation-dispersion regimes. Figure 15 presents the Sharpe ratio of long-only and trend-following strategies that make use of either VP or RP allocations.

Our main focus is the high-correlation/high-dispersion regime (red boxes). Clearly, the benefit of the risk-parity allocation that we uncovered in Figures 13 and 14 appears to be largely due to the benefit of the weighting scheme in periods of not just high correlation, but also high dispersion of pairwise correlations. The Sharpe ratio of a long-only allocation increases from 0.57 to 0.72 when switching the weighting scheme between VP and RP. The performance benefit is much more pronounced in the trend-following space with the respective Sharpe ratio more than doubling from 0.24 up to 0.56. No other correlation-dispersion regime exhibits similarly strong patterns.

Figure 15: Correlation – Dispersion Event Study



Source: UBS Quantitative Research. The figure presents the annualised Sharpe ratio of long-only and trend-following strategies that use either a volatility-parity or a risk-parity weighting scheme for four different combination states of average pairwise correlation and dispersion of pairwise correlations. The sample period is from April 1988 to December 2013.

A De-correlation Anomaly

The findings of the previous section can be summarised in two main points:

- Risk-parity allocations differ significantly from volatility-parity allocations in periods of high dispersion of pairwise correlations.
- In terms of strategy performance, whether long-only or trend-following, multi-asset-class strategies benefit from a risk-parity allocation mainly in periods of high correlation and high dispersion.

These findings can give rise to very important research questions.

Risk-parity…
- **Over-weights:**
  - **Low-volatility assets**
  - **De-correlated assets**
- **Under-weights:**
  - **High-volatility assets**
  - **Correlated assets**

By construction, risk-parity over-weights assets with lower volatility and/or assets that de-correlate with the rest of the universe and similarly under-weights assets with higher volatility and/or assets that correlate more with the universe. This is done in an effort to increase portfolio diversification, as already documented in Figure 3 at the beginning of this research note. However, given the outperformance of risk-parity long-only and trend-following strategies, it is worth exploring whether the increase in the risk-adjusted performance comes solely from optimising the downside of the portfolio return distribution or it also comes from extracting additional risk-adjusted returns from the over-weighted assets. In other words, as already outlined in our theoretical motivation section, we ask two more general research questions:

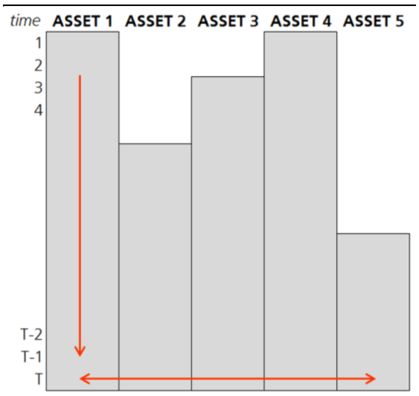
1. *Do <u>low-volatility</u> assets exhibit larger risk-adjusted returns in the cross-section?*
2. *Do <u>de-correlated</u> assets exhibit larger risk-adjusted returns in the cross-section?*

Low-risk anomaly is one of the reasons of risk-parity popularity

The answer to the first question is relatively well known following recent academic empirical work. Contrary to the predictions of Markowitz's (1952) modern portfolio theory and Sharpe's (1964) and Lintner's (1965) capital asset pricing theorem (CAPM), research has shown that low volatility assets (within and across asset classes, see Frazzini and Pedersen, 2014) tend to enjoy larger risk-adjusted returns in the cross-section. This empirical pattern is known as the low-risk or low-volatility anomaly and is definitely one of the reasons of the popularity and profitability of risk-parity strategies (e.g. see Asness, Frazzini and Pedersen, 2012).

However, the answer to the second question is not clear-cut and even though it is partly related to low-beta patterns it does need to be empirically tested. In this section we design a very broad panel regression setup and aim to answer both questions simultaneously for our universe of 35 assets across all asset classes.

Figure 16: Unbalanced Panel Dataset



Source: UBS Quantitative Research

First, we need to carefully appreciate the dataset that we have at hand. Failing to do so can result in a misspecified model or, if the standard errors of the regression are not properly adjusted, in over-rejection of the null hypotheses (and therefore in spurious findings). As illustratively depicted in Figure 16, our panel dataset is one of the hardest to work with in econometrics. This is due to a number of reasons:

- It is unbalanced, i.e. the number of monthly observations is not the same for each asset. In particular, we have 35 assets, with the shortest history lasting for 86 months and the longest lasting for 309 months (effectively covering the entire sample period, April 1988 to December 2013). Overall, we have 9797 return-month pairs of observations.
- At the asset level (vertical direction), we have serial correlation of returns, which is anyhow confirmed by the strong trend-following patterns that we document in our recent Global Quantitative Research Monograph "*Trend-*

following meets Risk-parity" (2 December 2013). A proper statistical test (Breusch-Godfrey test) applied to our panel dataset also confirms this.

- At the asset level again, we have heteroskedasticity patterns, i.e. the conditional volatility of the assets is time-varying. A proper statistical test (Breusch-Pagan test) confirms this.
- Across assets (horizontal direction), we have cross-sectional dependence, or in other words, assets tend to exhibit co-movement patterns. Proper statistical tests (Breusch-Pagan LM test and Pesaran CD test) confirm this.
- Finally, appropriately designed statistical tests cannot reject the existence of asset-specific – or asset-class-specific – fixed effects (using the Hausman test) as well as time fixed effects (using the F test). The existence of asset fixed effects means that each asset (or asset class) has a different unconditional mean return and the panel regression must be designed so that it includes a number of different intercepts/dummies (one per asset or per asset class) to accommodate this. The existence of time fixed effects means that all the asset returns are simultaneously affected by a market wide systematic signal (one can think of it as a cross-asset-class definition of "the market"); the panel regression must accommodate this.

Taking into account all the above features of our dataset, we construct and estimate the following panel regression that aims to identify the factors that explain the cross-sectional variation of the risk-adjusted returns of our 35 multi-asset-class universe:

$$\frac{r_{i,t}}{\sigma_{i,t}} = \underbrace{\alpha_i}_{\substack{asset\\FE}} + \underbrace{\beta_{mom} \cdot \frac{r_{i,t-1}^{mom}}{\sigma_{i,t-1}}}_{\substack{12-month\\momentum}} + \underbrace{\beta_\sigma \cdot \sigma_{i,t}}_{\substack{90-day\\volatility}} + \underbrace{\beta_{\Delta\rho} \cdot \Delta\rho_{i,t}}_{\substack{monthly\\change\ in\\correlation}} + \underbrace{\beta_{int} \cdot r_{i,t-1}^{mom} \cdot \rho_{i,t-1}}_{\substack{momentum-correlation\\interaction}} + \underbrace{\delta_t}_{\substack{time\\FE}} + u_{i,t} \quad (10)$$

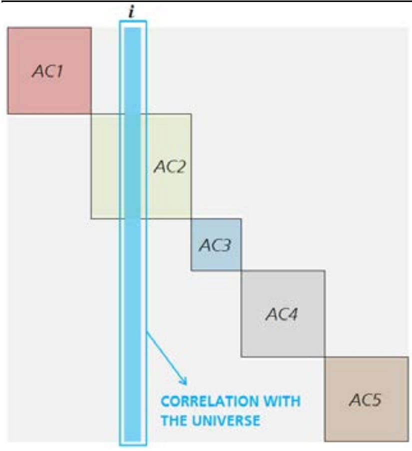
where:

- $\frac{r_{i,t}}{\sigma_{i,t}}$ is the risk-adjusted return of each asset at the end of each month. The risk-adjustment of the left hand side variable of the regression aims to alleviate the heteroskedasticity patterns.
- a_i is the asset-specific intercept that captures the asset fixed effect.
- $r_{i,t-1}^{mom}$ is the lagged 12-month return of each asset that captures the trend-following (serial correlation) patterns. The variable is scaled by the contemporaneous asset volatility to alleviate again any heteroskedasticity patterns.
- $\sigma_{i,t}$ is the 90-day realised volatility of each asset that aims to capture the risk-return relationship (and uncover any low-risk patterns)
- $\Delta\rho_{i,t}$ is the monthly change in the average correlation of asset i with the rest of the universe. In other words, $\rho_{i,t}$ is defined as the average value of the i^{th} column of the correlation matrix (see the blue box in Figure 17):

$$\rho_{i,t} = \frac{1}{N_t - 1} \sum_{\substack{j=1\\j\neq i}}^{N_t} \rho_{i,j,t} \quad (11)$$

This is the most important variable in our panel regression. It is of critical importance to highlight that this quantity is, on one hand, related to the correlation of an asset with some portfolio of all assets $\rho_{i,P}(\boldsymbol{w})$ as the one we

Figure 17: Correlation Matrix



Source: UBS Quantitative Research

introduced in equation (3), but the two measures are *not* identical. The former, as per equation (11), is the simple – equal weight – average of pairwise correlations, whereas the latter depends explicitly on the weighting scheme used for the construction of the portfolio.[8] The reason why we use the simple average is because we aim to focus on the core correlation effects and not swamp them with any weighting scheme.

- $r_{i,t-1}^{mom} \cdot \rho_{i,t-1}$ is the product of the lagged 12-month return and the lagged correlation of each asset with the rest of the universe. This interaction term will be used to explore whether there exists any empirical relationship between trend-following behaviour of an asset and its correlation with the rest of the universe.
- δ_t is the time fixed effect that aims to capture any systematic market-wide component of cross-sectional returns.

List of hypotheses for the panel regression

Before presenting the results of the regression, it is worth phrasing a list of hypotheses that would support or negate our expectations:

a) If $\beta_{mom} > 0$, then we have statistical evidence of trend-following.

b) If $\beta_{\sigma} > 0$, then higher-risk is rewarded with higher risk-adjusted return, in line with Markowitz's (1952) modern portfolio theory.

c) If (instead) $\beta_{\sigma} < 0$, then we have evidence of the low-volatility anomaly.

d) If $\beta_{\Delta\rho} > 0$, then higher co-movement with the rest of the universe is rewarded with higher risk-adjusted return, roughly in line with asset pricing principles (e.g. Cochrane, 2000).

e) If (instead) $\beta_{\Delta\rho} < 0$, then assets that get more de-correlated with the universe enjoy greater risk-adjusted returns in the cross-section.

f) If $\beta_{int} < 0$, then the de-correlated assets exhibit stronger trend-following patterns.

If hypotheses **(c) and (e)** hold true, then a risk-parity allocation is largely benefitted, because, on top of optimising portfolio diversification, it also promotes the best-performing assets in the cross-section. If hypotheses **(a) and (f)** hold true, then a risk-parity allocation is also the better scheme to use in a trend-following portfolio, as the assets that it promotes would tend to exhibit stronger trend-following patterns.

Figure 18 presents the results of the panel regression with the t-statistics that are reported being calculated using non-parametric Driscoll and Kraay (1998) standard errors that account for heteroskedasticity, cross-sectional and serial correlation.[9,10]

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[8] Dropping, for the sake of illustration, the time dependence in equation (11), the average correlation of asset i with the rest of the universe and the correlation of the same asset with a portfolio of all assets are shown below:

$$\rho_i = \sum_{\substack{j=1 \\ j \neq i}}^{N} \frac{1}{N-1} \cdot \rho_{i,j} \text{ versus } \rho_{i,P} = \sum_{j=1}^{N} w_j \cdot \frac{\sigma_j}{\sigma_P} \cdot \rho_{i,j}$$

[9] Accounting properly for the various statistical features of the dataset is of critical importance in the estimation of standard errors and subsequently t-statistics. Using simple OLS standard errors largely reduces the standard errors and therefore largely increases the t-statistics leading to false inference.

[10] Instead of the non-parametric Driscoll and Kraay (1998) standard errors, one could use time-clustered standard errors using the heteroskedasticity and autocorrelation-consistent covariance matrix by Arellano (1987). We have confirmed that the resulting standard errors are very close numerically and our results remain unaltered.

Figure 18: Panel Regression for explaining cross-sectional risk-adjusted returns

	(1)	(2)	(3)	(4)	(5)	(6)
$\boldsymbol{\beta_{mom}} \cdot \mathbf{100}$	2.03			2.08	2.32	2.39
	(4.61)			**(4.72)**	**(4.37)**	**(4.54)**
$\boldsymbol{\beta_{\sigma}} \cdot \mathbf{100}$		-0.22		-2.01		-2.11
		(-0.04)		(-0.36)		(-0.38)
$\boldsymbol{\beta_{\Delta\rho}}$			-0.44	-0.45		-0.46
			(-2.63)	**(-2.71)**		**(-2.75)**
$\boldsymbol{\beta_{int}}$					-0.13	-0.14
					(-1.00)	(-1.12)
Adj. $\boldsymbol{R^2}$ (%)	0.61	0.00	0.24	0.86	0.63	0.89

Source: UBS Quantitative Research. The table presents the panel regression results from regressing the risk-adjusted returns of the assets on a list of cross-sectional variables. The t-statistics are calculated using non-parametric Driscoll and Kraay (1998) standard errors that account for heteroskedasticity, cross-sectional and serial correlation. The sample period is from April 1988 to December 2013.

The results are insightful. Commenting on a row-by-row basis, the results first show that the trend-following patterns are statistically very strong, in line with the results of our Global Quantitative Research Monograph "*Trend-following meets Risk-Parity*" (2 December 2013). Contrary to the predictions of the CAPM, higher risk is *not* compensated by higher returns, as the coefficient β_σ is negative, even though statistically insignificant, hence adding some support to the long list of academic papers that document the low-volatility anomaly. It is important to highlight that we document these patterns on a cross-asset-class framework and not just on the equity space that has been the main playground of low-volatility strategies.

Assets that de-correlate exhibit larger risk-adjusted returns

Moving on to the most important finding of this research note, the coefficient $\beta_{\Delta\rho}$ is not just negative, but it's also strongly statistically significant at the 1% level irrespective of the specification. This means that assets that de-correlate with the universe over the most recent month enjoy larger risk-adjusted returns in the cross-section. This is a novel finding, especially on a multi-asset-class universe.

Demystifying low-beta anomaly

One could relate this finding to the low-beta anomaly, which, even though related to low-volatility, it should be thought of as a more general phenomenon. To explain this better, the beta of an asset is trivially equal to its volatility times its correlation with some definition of the market (trivially defined at the equity space, more vague in a multi-asset-class universe), all scaled by the market volatility, σ_M:

$$\beta_i = \frac{Cov(r_i, r_M)}{\sigma_M^2} = \frac{\sigma_i}{\sigma_M} \cdot \rho_{i,M} \qquad (12)$$

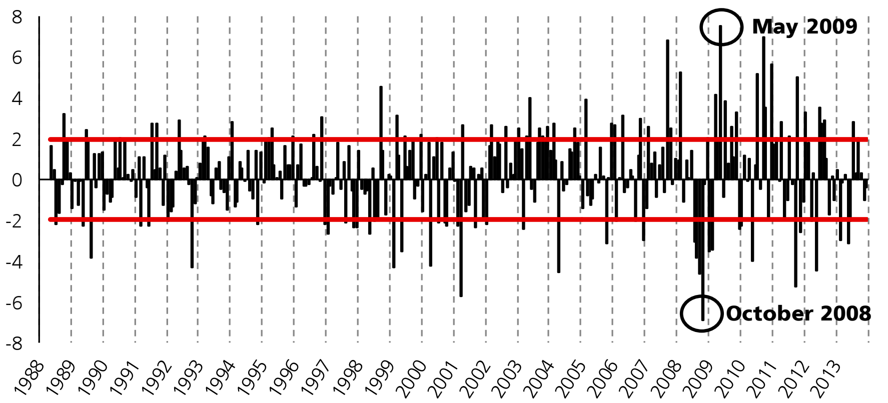
The beta of an asset can be cross-sectionally low, because either its volatility is cross-sectionally low or its correlation with the market is low or both. In other words, a low-beta effect can either be a low-volatility effect, a de-correlation effect or a combination of the two. What we have shown with the results of the panel regression in Figure 18 is that the de-correlation channel is statistically much stronger than the low-volatility channel in a multi-asset-class universe.

Continuing with the results, the point estimate of the coefficient of the momentum-correlation interaction term β_{int} is negative, but statistically insignificant. Interpreting the sign of the coefficient we can argue that assets which have lower correlation with the rest of the universe tend to exhibit stronger

trend-following patterns. Even though not statistically strong, this finding acts in favour of a risk-parity allocation when applied to trend-following strategies.

The final piece of empirical evidence from the panel regression relates to the identified time fixed effect δ_t. As already mentioned, this component captures any systematic market-wide component of asset returns across all asset classes. Figure 19 presents the t-statistic of the time fixed effect at the end of each month in an effort to comprehend the time evolution of significance of this market-wide systematic shock. The big majority of significant spikes start after September 2007 with the largest negative spike being in October 2008, in the month following the collapse of Lehman Brothers and the largest positive spike being in May 2009 when the global equity markets rebounded from the financial crisis drawdowns.

Figure 19: Time Fixed Effect: t-statistic

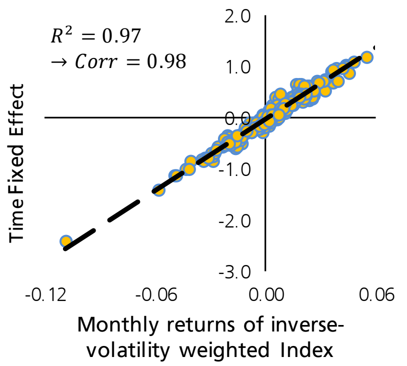


Source: UBS Quantitative Research. The figure presents the t-statistic of the time fixed effect δ_t of the panel regression (10). The red straight lines at +1.96 and -1.96 identify the 5% confidence levels. The sample period is from April 1988 to December 2013.

These findings constitute evidence that our regression setup is correctly specified as it can reasonably capture the behaviour of some cross-asset-class definition of "the market". To investigate this further, we explore the correlation structure between the extracted time fixed effect and an appropriately designed "market" index for the cross-asset-class universe. The most reasonable choice for such an index would seem to be the inverse-volatility weighted portfolio of all futures contracts[11].

Figure 20 presents a scatterplot between the time fixed effect of the panel regression and the monthly returns of the inverse-volatility market index. The result is unambiguous. The extracted time fixed effect from the panel regression is almost identical to a cross-asset-class market index with the correlation being close to perfect (0.98). We couldn't have asked for a stronger sanity test of our methodology.

Figure 20: Time Fixed Effect vs. an Inverse-Volatility Weighted Index



Source: UBS Quantitative Research. The figure presents a scatterplot between the time fixed effect δ_t of the panel regression (10) and the monthly returns of the inverse-volatility weighted index of all assets in our universe. The sample period is from April 1988 to December 2013.

[11] Unsurprisingly, this inverse-volatility weighted index coincides with the unlevered volatility-parity strategy that has been extensively studied in our Global Quantitative Research Monograph "*Trend-Following meets Risk Parity*" (2 December 2013) and our Quant Keys "*Risk-Parity versus Mean-Variance*" (16 May 2014).

Which part of correlation matters the most?

Given that our universe consists of 35 futures contracts across all asset classes, the average correlation of an asset i with the rest of the universe, $\rho_{i,t}$, can be decomposed into two components: the correlation part that relates to the assets of the same asset class (intra-asset-class correlation) and the correlation part that related to the assets of all the other asset classes (inter-asset-class correlation). Along these lines, equation (8) can be rewritten as follows (see also Figure 21):

$$\rho_{i,t} = \frac{1}{N_t - 1}\left[\sum_{\substack{j\in AC(i)\\ j\neq i}} \rho_{i,j,t} + \sum_{j\notin AC(i)} \rho_{i,j,t}\right] \qquad (13)$$

where $AC(i)$ denotes the set of assets that belong to the asset class of asset i. Following this decomposition, we can define for each asset the average intra-asset-class correlation and the average inter-asset-class correlation as follows:

$$\rho_{i,t}^{Intra} = \frac{1}{N_{AC(i),t} - 1}\sum_{\substack{j\in AC(i)\\ j\neq i}} \rho_{i,j,t} \qquad (14)$$

and

$$\rho_{i,t}^{Inter} = \frac{1}{N_t - N_{AC(i),t}}\sum_{j\notin AC(i)} \rho_{i,j,t} \qquad (15)$$

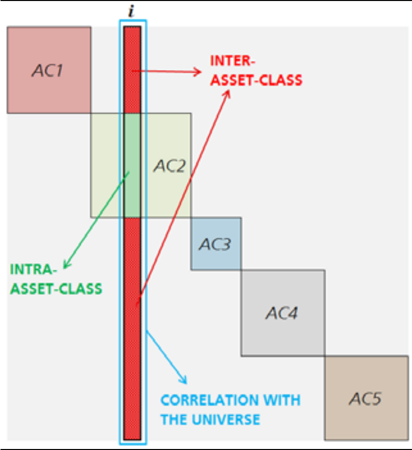
where $N_{AC(i),t}$ denotes the number of assets that belong in the asset class of asset i at time t. Notice that $\rho_{i,t} \neq \rho_{i,t}^{Intra} + \rho_{i,t}^{Inter}$, unless $N_{AC(i),t} = (N_t + 1)/2$.

Using the above two definitions of the two correlation components, we can extend the panel regression setup of equation (10) and introduce two terms that can disentangle the significant relationship between asset risk-adjusted returns and their correlation with the universe into intra and inter-asset-class effects:

$$\frac{r_{i,t}}{\sigma_{i,t}} = \alpha_i + \beta_{mom}\cdot\frac{r_{i,t-1}^{mom}}{\sigma_{i,t-1}} + \beta_\sigma\cdot\sigma_{i,t} + \underbrace{\beta_{\Delta\rho}^{Intra}\cdot\Delta\rho_{i,t}^{Intra}}_{\substack{monthly\ change\ in\\ intra-asset-class\\ correlation}} + \underbrace{\beta_{\Delta\rho}^{Inter}\cdot\Delta\rho_{i,t}^{Inter}}_{\substack{monthly\ change\ in\\ inter-asset-class\\ correlation}} + \beta_{int}\cdot r_{i,t-1}^{mom}\cdot\rho_{i,t-1} + \delta_t + u_{i,t} \qquad (16)$$

It is an important research question to identify whether the significant return-correlation relationship is due to the correlation profile of the assets within their asset class group or outside of it or even a combination of the two. Figure 22 augments the results of Figure 18 by incorporating these two new correlation terms (focus on the green and red boxed rows).

Figure 21: Correlation Matrix



Source: UBS Quantitative Research

Intra-asset-class correlations do NOT matter

Inter-asset-class correlations DO matter

The results are enlightening. Intra-asset-class correlations have no significant effect in the cross-section of asset returns (even though the sign of the point estimates is negative in line with the coefficient of $\beta_{\Delta\rho}$). Instead, it is mainly the intertemporal variability of asset correlation with the assets of the other asset classes that captures the documented negative return-correlation relationship.

Figure 22: Panel Regression for explaining cross-sectional risk-adjusted returns: Splitting the correlation effects

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
$\beta_{mom} \cdot 100$	2.03						2.08	2.09	2.32	2.39	2.40
	(4.61)						(4.72)	(4.72)	(4.37)	(4.54)	(4.55)
$\beta_{\sigma} \cdot 100$		-0.22					-2.01	-2.00		-2.11	-2.10
		(-0.04)					(-0.36)	(-0.36)		(-0.38)	(-0.38)
$\beta_{\Delta\rho}$			-0.44				-0.45			-0.46	
			(-2.63)				(-2.71)			(-2.75)	
$\beta_{\Delta\rho}^{Intra}$				-0.16		-0.10		-0.10			-0.10
				(-1.84)		(-1.33)		(-1.34)			(-1.34)
$\beta_{\Delta\rho}^{Inter}$					-0.39	-0.35		-0.35			-0.35
					(-2.64)	(-2.52)		(-2.59)			(-2.63)
β_{int}									-0.13	-0.14	-0.14
									(-1.00)	(-1.12)	(-1.12)
Adj. R^2 (%)	0.61	0.00	0.24	0.09	0.22	0.25	0.86	0.87	0.63	0.89	0.89

Source: UBS Quantitative Research. The table presents the panel regression results from regressing the risk-adjusted returns of the assets on a list of cross-sectional variables. The t-statistics are calculated using non-parametric Driscoll and Kraay (1998) standard errors that account for heteroskedasticity, cross-sectional and serial correlation. The sample period is from April 1988 to December 2013.

A note on statistical robustness and R^2s of panel regressions

Estimating a panel regression is always a challenge especially when the panel is unbalanced and serial-correlation and heteroskedasticity effects come in to play like in our case. Our mainstream setup of equations (10) and (16) risk-adjusts left and right hand side return variables to alleviate some of these concerns. An alternative is to use the z-score of the variable, where the normalising is applied at the asset level before the return series of the various assets are pooled together for the estimation of the regression. The results from this alternative specification (see Appendix E) do not dramatically change our main finding that changes in correlations and in particular changes in inter-asset-class correlations largely determine the cross-sectional variation of asset returns.

Another alteration that could be applied in the panel regression setup is to use asset class fixed effects instead of individual asset fixed effects. This would effectively reduce the intercepts in equations (10) and (16) from 35 down to 5. Given the relatively large similarity between contracts of the same asset class, our main finding remains again unchanged and we therefore omit these results from the report.

Finally, we need to make a note on the level of the R^2s in these panel regressions, because they appear to have relatively low values (<1%). However, this is by no means worrying about the credibility or validity of our results, as the calculation of the R^2 of a panel regression resembles that of a cross-sectional regression and it is relatively well known in the field of econometrics and financial economics that the R^2s of cross-sectional regressions are in general low (contrary to time-series R^2s). The technicalities of this argument go beyond the scope of this short comment on R^2s, but in simple illustrative terms, a panel regression R^2 is somewhat similar to an average of cross-sectional R^2s at each point in time (at the end of each month in our case), rather than an average of individual time-series R^2s.

Intra and Inter-Asset-Class Correlations

Following the results of the panel regressions, we investigate further how the pairwise correlations within and across asset classes have evolved over time. Figure 23 illustrates the various components of a generic multi-asset-class correlation matrix for which we estimate 90-day average pairwise correlations and present them in Figure 24. Some very interesting patterns emerge:

Intra-asset-class level: Except for commodities, pairwise correlations at the intra-asset-class level have been historically fluctuating at elevated levels, without strong trends (with the possible exception of equities that demonstrate a slow but smooth increase over the last 20 years, most probably due to the larger degree of global equity market integration). The average pairwise correlations exhibit in general a mean reverting behaviour around a non-zero long term mean which is roughly around 50% for energy, 40% for fixed income and 30% for FX and equities.

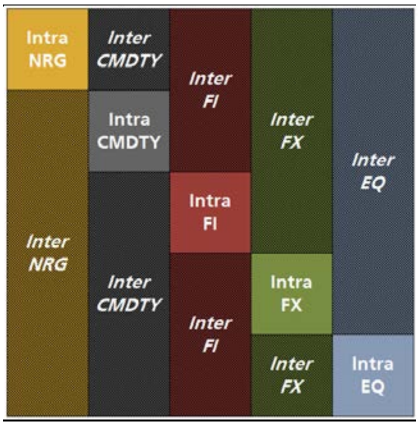
Contrary to these patterns, commodities have exhibited significantly lower pairwise correlations (around 15%) and this is why they have always been regarded as the most diversified universe (e.g. see Erb and Harvey, 2006 and Gorton and Rouwenhorst, 2006). This diversifying nature has been however largely distorted after 2004, when the asset class of commodities has started becoming more integrated (e.g. see Cheng, Kirilenko and Xiong, 2014) reaching levels of average pairwise correlation of up to 50% at the end of 2008 following the credit crisis. This is clearly the most interesting finding of the intra-asset-class level analysis.

For the sake of completeness, the documented co-movement patterns within commodities over the last decade have been claimed to be the result of two phenomena that are jointly referred to as the "*financialisation of commodities*": on one hand, the introduction of the Commodity Futures Modernization Act (CFMA) in 2000 allowed investors to hedge commodity price risk using futures contracts and on the other hand, commodities were packaged in indices like the S&P GSCI index or the Dow Jones-UBS Commodity index. Both effects can generate co-moving mechanisms across fundamentally different commodities. This has been a very active research field recently.[12]

Inter-asset-class level: The patterns of average correlations across asset classes are far more interesting and, surprisingly, they have rarely been studied in the literature. As shown in Figure 24, for each asset class, the average pairwise correlations with all the other asset classes have been fairly low (if not insignificant), roughly fluctuating between -0.20 and 0.20 until the end of 2003.

However, post 2004, asset classes have started becoming much more significantly correlated, which of course has dramatically hurt the diversification benefits.[13] Across all asset classes, the inter-asset-class average pairwise correlation has significantly increased since 2004 and has remained at higher-than-historical levels since then. The only very important exception has been the fixed income asset class, which, even though it initially followed the increasing correlation patterns, it then returned back to lower levels in 2007 and has remained there ever since.

Figure 23: Correlation & Asset Class

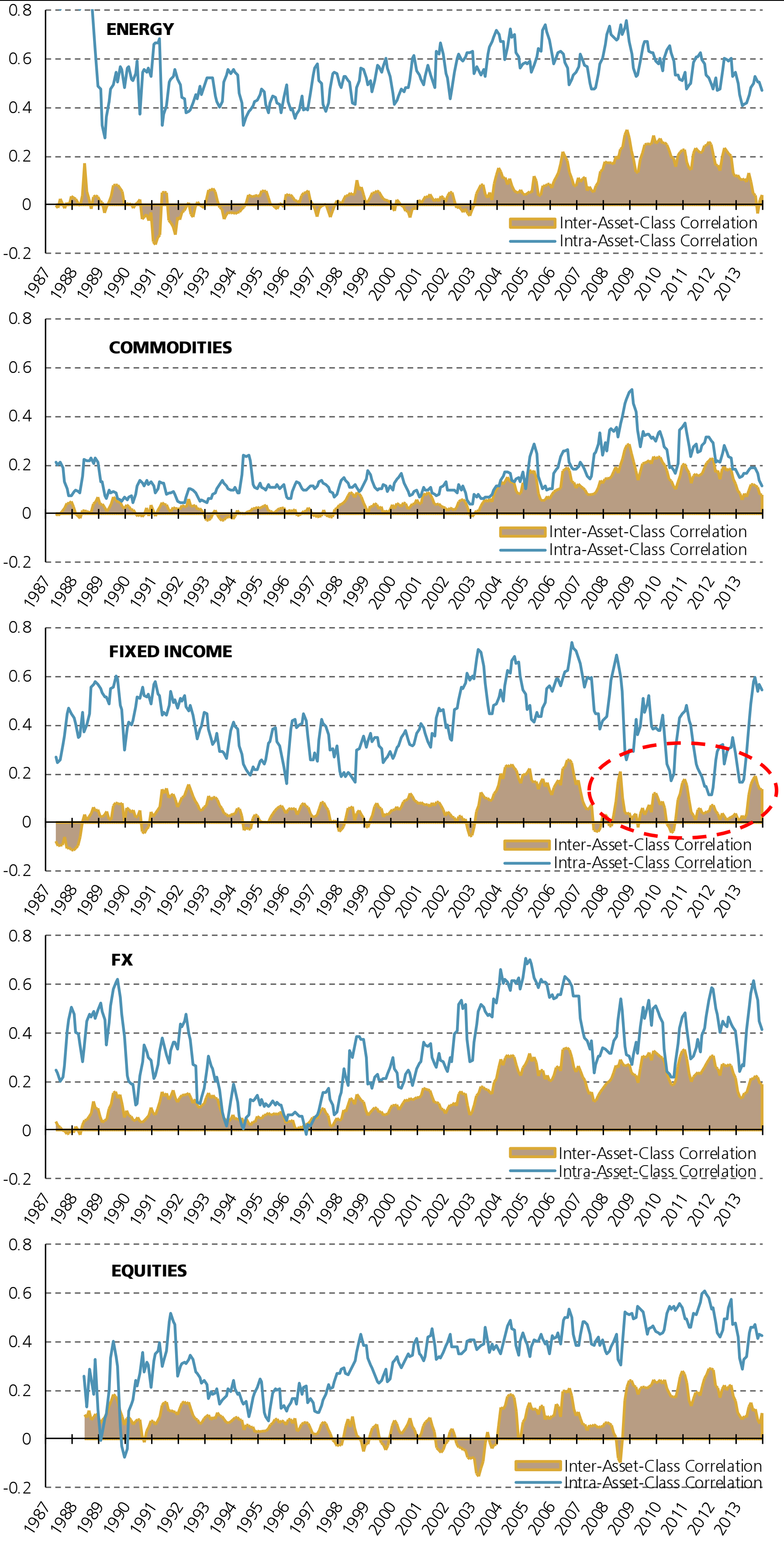


Source: UBS Quantitative Research. The figure illustrates the parts of the correlation matrix that are used for the calculation of the intra and inter asset class average pairwise correlations of Figure 24.

[12] Indicatively, see the papers by Falkowski (2011), Irwin and Sanders (2011), Tang and Xiong (2012), Basak and Pavlova (2013), Henderson, Pearson and Wang (2013) and Cheng and Xiong (2014) as well as references therein.

[13] See our Global Quantitative Research Monograph "*All Together Now*" (19 August 2010).

Figure 24: Intra and Inter Asset-Class Average Pairwise Correlations



Source: UBS Quantitative Research. 90-day average pairwise correlation at the inter and intra asset-class level.

This finding clearly indicates that the asset class of fixed income has behaved relatively differently to the rest of asset classes over and after the recent financial crisis and one could heuristically argue that we can split the investable universe into two hyper-groups: the fixed income assets and the non-fixed income assets. This distinction could also be related to the "*Risk-On*" and "*Risk-Off*" terminology that has been heavily used recently.

Fixed income and the rest:

➔ Risk-On, Risk-Off

Following the above observations, as well as client interest, it is rather challenging to ask whether our results hold if we were to exclude the fixed income assets from our universe. One could argue that the outperformance of the low-volatility and de-correlated (with the rest of the universe) fixed income assets over the last decade can be the main reason for the identification of the patterns in our panel regression setup. Low-volatility and risk-parity strategies have recently received similar type of criticism in that their strong historical outperformance is largely due to their large bias on bonds (see e.g. Inker, 2011, Ruban and Melas, 2011).

Are our results mainly driven by fixed income assets?

Figure 25 replicates Figure 22 after excluding from the universe the 6 fixed income contracts, eventually leading to 29 contracts in total. The evidence is very convincing; our main conclusions remain qualitatively unchanged. Even if the statistical significance of the correlation variables drops slightly, $\beta_{\Delta\rho}$ and most importantly $\beta_{\Delta\rho}^{Inter}$ remain statistically strong hence supporting that the de-correlation patterns are not just an artefact of the recent outperformance of the de-correlated (with the rest of the universe) fixed-income assets but the effects are pervasive across asset classes.

Figure 25: Panel Regression after excluding the Fixed Income Assets

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
$\boldsymbol{\beta_{mom}} \cdot \mathbf{100}$	2.21						2.23	2.24	2.54	2.58	2.59
	(5.41)						**(5.48)**	**(5.49)**	**(5.13)**	**(5.23)**	**(5.25)**
$\boldsymbol{\beta_{\sigma}} \cdot \mathbf{100}$		3.26					0.93	0.94		0.84	0.85
		(0.61)					(0.18)	(0.18)		(0.16)	(0.17)
$\boldsymbol{\beta_{\Delta\rho}}$			-0.29				-0.31			-0.31	
			(-1.94)				**(-2.08)**			**(-2.12)**	
$\boldsymbol{\beta_{\Delta\rho}^{Intra}}$				-0.14		-0.10		-0.10			-0.10
				(-1.57)		(-1.19)		(-1.18)			(-1.19)
$\boldsymbol{\beta_{\Delta\rho}^{Inter}}$					-0.26	-0.22		-0.24			-0.24
					(-2.05)	**(-1.88)**		**(-2.03)**			**(-2.06)**
$\boldsymbol{\beta_{int}}$									-0.12	-0.12	-0.12
									(-1.33)	(-1.38)	(-1.39)
Adj. R^2 (%)	0.72	0.01	0.11	0.06	0.11	0.14	0.85	0.88	0.75	0.88	0.91

Source: UBS Quantitative Research. The table replicates the results of Figure 22 after excluding the fixed income assets from the universe. The sample period is from April 1988 to December 2013.

In order to further support the above arguments, we re-run the analysis by excluding one asset class at a time and report in Figure 26 the columns (10) and (11) of Figure 25 for each case (the results for all assets and for the exclusion of fixed income assets have already been presented in Figure 22 and Figure 25 respectively, but are also included here for comparison and completeness).

Figure 26: Panel Regression after excluding one asset class at a time

	ALL ASSETS		ex. ENERGY		ex. COMMODITIES		ex. FIXED INCOME		ex. FX		ex. EQUITIES	
$\beta_{mom} \cdot 100$	2.39	2.40	2.35	2.34	2.94	2.94	2.58	2.59	2.16	2.16	2.24	2.25
	(4.54)	**(4.55)**	**(3.90)**	**(3.88)**	**(4.05)**	**(4.06)**	**(5.23)**	**(5.25)**	**(3.78)**	**(3.78)**	**(4.26)**	**(4.28)**
$\beta_{\sigma} \cdot 100$	-2.11	-2.10	-9.08	-9.23	-3.60	-3.55	0.84	0.85	-0.08	-0.10	2.14	2.18
	(-0.38)	(-0.38)	(-1.36)	(-1.38)	(-0.53)	(-0.52)	(0.16)	(0.17)	(-0.01)	(-0.02)	(0.32)	(0.33)
$\beta_{\Delta\rho}$	-0.46		-0.47		-0.33		-0.31		-0.57		-0.44	
	(-2.75)		**(-3.05)**		**(-1.64)**		**(-2.12)**		**(-3.38)**		**(-2.40)**	
$\beta_{\Delta\rho}^{Intra}$		-0.10		-0.05		-0.10		-0.10		-0.13		-0.17
		(-1.34)		(-0.56)		(-1.07)		(-1.19)		(-1.33)		**(-1.86)**
$\beta_{\Delta\rho}^{Inter}$		-0.35		-0.38		-0.24		-0.24		-0.43		-0.27
		(-2.63)		**(-3.42)**		(-1.50)		**(-2.06)**		**(-3.27)**		**(-1.84)**
β_{int}	-0.13	-0.14	-0.25	-0.24	-0.19	-0.19	-0.12	-0.12	-0.09	-0.08	-0.16	-0.16
	(-1.12)	(-1.12)	(-1.56)	(-1.56)	(-1.09)	(-1.10)	(-1.38)	(-1.39)	(-0.66)	(-0.65)	(-1.33)	(-1.33)
Adj. R^2 (%)	0.89	0.89	0.84	0.83	1.18	1.19	0.88	0.91	0.91	0.91	0.73	0.75

Source: UBS Quantitative Research. The table replicates the results of Figure 22 after excluding one asset class from the universe at a time. The sample period is from April 1988 to December 2013.

Our results remain unchanged no matter which asset class we exclude from the universe

Our results remain, by and large, qualitatively unchanged. The exclusion of commodities seems to be the only scenario that results in a marginal loss of the statistical significance of our inter-asset-class findings, but overall we consider the negative return-correlation relationship rather robust.

Overall, this section documents a very novel finding. On a multi-asset-class framework, the assets that de-correlate more with the rest of the universe tend to enjoy larger risk-adjusted returns. It's not the low-volatility channel, but more importantly the low-correlation channel that is driving the patterns.

Conclusion

The aim of this research note was to touch upon two general topics related to risk-parity investing using a cross-section of 35 futures contracts across all asset classes.

First, we investigated the reasons why a risk-parity scheme deviates from a standard inverse-volatility scheme. Clearly, pairwise correlations between the portfolio constituents are largely controlling this relationship. However, contrary to our expectations that the average level of correlation is the dominant factor of this relationship, we found that it is in fact the way by which all pairwise correlations are dispersed around this average level that determines how different a risk-parity portfolio is from an inverse-volatility scheme.

Importantly enough, the benefit of a risk-parity allocation in terms of performance is mostly pronounced in the states of the market when both average correlation and correlation dispersion are high.

Second, motivated by the recent outperformance of risk-parity schemes, we explored a more fundamental research question relating to the asset pricing implications that can be related to this outperformance. Contrary to theoretical expectations that instruct larger expected returns for assets that co-vary with the overall market, we instead found that assets which de-correlate with the rest of the universe enjoy greater risk-adjusted returns. We termed this phenomenon as the de-correlation anomaly, which interestingly sits above and beyond the very-well known low-volatility anomaly.

Future direction of research

Our results have important implications for portfolio construction and asset allocation. Future theoretical research should address the reasons why the de-correlation anomaly is so prevalent in a multi-asset-class universe; existing explanations of the low-volatility/low-risk anomaly could very well be good candidate explanations here (see Baker, Bradley and Wurgler, 2011, Asness, Frazzini and Pedersen, 2012, Frazzini and Pedersen, 2014 as well as our Q-Series® "*Low-Risk Investing*", 23 September 2011).

APPENDIX

A. Overview of the Dataset

Figure 27 contains the list of the 35 futures contracts that we use in this note. The data is retrieved from Bloomberg and contains daily prices of the generic ratio-backwards adjusted continuous-price series. The dataset is identical to the one used in our recent Quant Keys "*Risk-Parity versus Mean-Variance*" (16 May 2014).

Figure 27: Dataset

ENERGY		COMMODITIES		FIXED INCOME		FX		EQUITIES	
Brent Crude	Jul-88	Coffee "C"	Jan-87	German Bobl 5Yr	Nov-91	AUD	Feb-87	Dax	Dec-90
Gas Oil	Aug-89	Copper	Jan-89	German Bund 10Yr	Dec-90	CAD	Jan-87	EuroStoxx 50	Jul-98
Gasoline	Nov-05	Corn	Jan-87	Japanese GB 10Yr	Jan-87	CHF	Jan-87	FTSE 100	Mar-88
Heating Oil #2	Jan-87	Cotton #2	Jan-87	US T-Notes 5Yr	Jun-88	EUR	Jun-98	Kospi 200	Jun-96
Light Crude	Jan-87	Gold (100 oz.)	Jan-87	US T-Notes 10Yr	Jan-87	GBP	Jan-87	Nasdaq Composite	May-96
Natural Gas	May-90	Live Cattle	Jan-87	US T-Notes 30Yr	Jan-87	JPY	Jan-87	Nikkei	Oct-88
		Silver	Jan-87					S&P 500	Jan-87
		Sugar #11	Jan-87						
		Wheat	Jan-87						

Source: Bloomberg. The table reports the futures contracts that we use including the first month that each data series is available. The sample period reached December 2013.

B. Marginal Contribution to Risk

The marginal contribution to risk (MCR) of asset i is generally defined as follows:

$$MCR_i = \frac{\partial \sigma_P(\boldsymbol{w})}{\partial w_i} = \frac{(\boldsymbol{\Sigma} \cdot \boldsymbol{w})_i}{\sigma_P(\boldsymbol{w})} \quad (17)$$

where $(\cdot)_i$ denotes the i^{th} element of a vector. The nominator of the above expression is effectively the weighted sum of covariance terms of asset i with the rest of the universe, which can also be interpreted as the covariance of asset i with the entire portfolio, denoted by $\sigma_{i,P}$:

$$MCR_i = \frac{\sum_{j=1}^{N} w_j \cdot \sigma_{i,j}}{\sigma_P(\boldsymbol{w})} = \frac{\sigma_{i,P}(\boldsymbol{w})}{\sigma_P(\boldsymbol{w})} \quad (18)$$

Given that a covariance is mathematically equal to the product between correlation and volatilities, we can deduce that (see also Menchero and Davis, 2011):

$$MCR_i = \frac{\sigma_i \cdot \sigma_P(\boldsymbol{w}) \cdot \rho_{i,P}(\boldsymbol{w})}{\sigma_P(\boldsymbol{w})} = \sigma_i \cdot \rho_{i,P}(\boldsymbol{w}) \quad (19)$$

which proves the validity of equation (3). For the sake of completeness, equation (19) can give rise to an expression for the MCR that involves the beta of the asset with the overall portfolio, $\beta_{i,P}(\boldsymbol{w})$ (see also Lee, 2011 for a similar derivation):

$$MCR_i = \sigma_P(\boldsymbol{w}) \cdot \frac{\sigma_{i,P}(\boldsymbol{w})}{\sigma_P^2(\boldsymbol{w})} = \sigma_P(\boldsymbol{w}) \cdot \beta_{i,P}(\boldsymbol{w}) \quad (20)$$

C. Volatility-Parity versus Risk-Parity

As outlined in the theoretical section of this research paper, volatility-parity allocates weights that are inversely proportional to the volatilities of the assets:

$$w_i^{VP} \propto \frac{1}{\sigma_i} \tag{21}$$

Conversely, risk-parity allocates weights that are inversely proportional to the volatilities of the assets *and* their correlation with the overall portfolio:

$$w_i^{RP} \propto \frac{1}{MCR_i} = \frac{1}{\sigma_i} \cdot \frac{1}{\rho_{i,P}(\boldsymbol{w}^{RP})} \tag{22}$$

The factors of proportionality (FP, henceforth) in the above equations are determined using the "fully-invested" constraint, i.e. $\sum_{j=1}^{N} w_j = 1$. Trivially:

$$FP^{VP} = \frac{1}{\sum_{j=1}^{N} \frac{1}{\sigma_j}} \tag{23}$$

and

$$FP^{RP}(\boldsymbol{w}^{RP}) = \frac{1}{\sum_{j=1}^{N} \frac{1}{\sigma_j} \cdot \frac{1}{\rho_{j,P}(\boldsymbol{w}^{RP})}} \tag{24}$$

One very important comment here is that the factor of proportionality for VP is only dependent on the volatilities of the assets, whereas the factor of proportionality for RP is directly related to the weighting scheme (through the correlation terms $\rho_{j,P}(\boldsymbol{w}^{RP})$ in equation (24) above). Hence, it is determined in conjunction with the weighting scheme, so that the RP objective of equation (2) is achieved. This is the reason for the explicit dependence on the weight vector $\boldsymbol{w}^{RP}$ in our notation above.

Following the above, we can now elaborate further on the spread between VP and RP weights, which anyway relates to the root mean square deviation between the two weight vectors as presented in equation (7):

$$w_i^{VP} - w_i^{RP} = \frac{FP^{VP}}{\sigma_i} - \frac{FP^{RP}(\boldsymbol{w}^{RP})}{\sigma_i \cdot \rho_{i,P}} \tag{25}$$

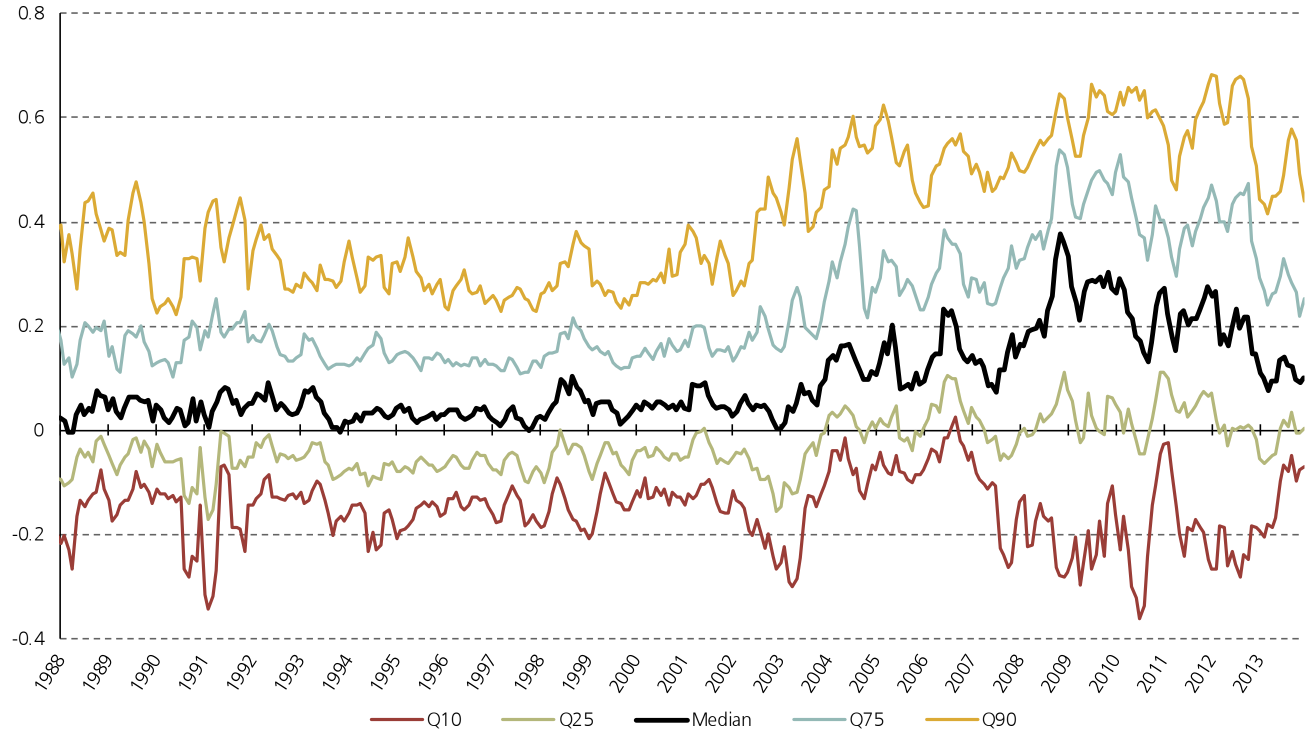
$$\Leftrightarrow w_i^{VP} - w_i^{RP} = \frac{1}{\sigma_i}\left[FP^{VP} - \frac{FP^{RP}(\boldsymbol{w}^{RP})}{\rho_{i,P}(\boldsymbol{w}^{RP})}\right] \tag{26}$$

Notice that when all pairwise correlations are the same, say equal to ρ, then $FP^{RP} = \rho \cdot FP^{VP}$. At the same time, it follows trivially that the correlation of each asset with the overall portfolio is also equal to ρ and therefore (27) results in equality between the two weighting schemes. This convergence was also proven using a slightly different methodology in our recent Quant Keys "*Risk-Parity versus Mean-Variance*" (16 May 2014).

D. Distribution of Pairwise Correlations

Figure 28 presents the time-series of certain percentiles of the cross-sectional distribution of 90-day pairwise correlations between all the assets of our universe. The cross-sectional average and standard deviation ("dispersion") that are presented in Figure 9, are the key quantities in the first part of this research report.

Figure 28: Cross-sectional Percentiles of Pairwise Correlations



Source: UBS Quantitative Research. The figure plots the 10th, 25th, 50th (median), 75th and 90th percentile of the pairwise correlations across all futures contracts at the end of each month (pairwise correlations are estimated using a 90-day estimation window). The sample period is from January 1988 to December 2013.

E. Robustness Results on Panel Regressions

This section of the appendix presents the results to an alternative specification of the panel regressions (10) and (16). In particular, instead of normalising the return-based variables with asset volatilities, we use the the z-score of the return and volatility variables. The normalising is applied at the asset level before the series of the various assets are pooled together for the estimation of the regression. The alternative to equations (10) and (16) are shown below, followed by the regression results in Figure 29.

$$\mathbf{Z}(r_{i,t}) = \underbrace{\alpha_i}_{\substack{asset\\ FE}} + \underbrace{\beta_{mom} \cdot \mathbf{Z}\left(r_{i,t-1}^{mom}\right)}_{\substack{12-month\\ momentum}} + \underbrace{\beta_{\sigma} \cdot \mathbf{Z}\left(\sigma_{i,t}\right)}_{\substack{90-day\\ volatility}} + \underbrace{\beta_{\rho} \cdot \Delta\rho_{i,t}}_{\substack{monthly\\ change\, in\\ correlation}} + \underbrace{\beta_{int} \cdot \mathbf{Z}\left(r_{i,t-1}^{mom}\right) \cdot \rho_{i,t-1}}_{\substack{momentum-correlation\\ interaction}} + \underbrace{\delta_t}_{\substack{time\\ FE}} + u_{i,t} \tag{27}$$

$$\mathbf{Z}(r_{i,t}) = \alpha_i + \beta_{mom} \cdot \mathbf{Z}\left(r_{i,t-1}^{mom}\right) + \beta_{\sigma} \cdot \mathbf{Z}\left(\sigma_{i,t}\right) + \underbrace{\beta_{\rho}^{Intra} \cdot \Delta\rho_{i,t}^{Intra}}_{\substack{monthly\ change\ in\\ intra-asset-class\\ correlation}} + \underbrace{\beta_{\rho}^{Inter} \cdot \Delta\rho_{i,t}^{Inter}}_{\substack{monthly\ change\ in\\ inter-asset-class\\ correlation}} + \beta_{int} \cdot \mathbf{Z}\left(r_{i,t-1}^{mom}\right) \cdot \rho_{i,t-1} + \delta_t + u_{i,t} \tag{28}$$

Figure 29: Panel Regression for explaining cross-sectional z-scores of returns

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
$\beta_{mom} \cdot 100$	3.05						3.24	3.25	4.20	4.51	4.52
	(1.39)						(1.47)	(1.47)	**(1.69)**	**(1.83)**	**(1.84)**
$\beta_{\sigma} \cdot 100$		-0.49					-0.77	-0.77		-0.72	-0.72
		(-0.23)					(-0.36)	(-0.36)		(-0.34)	(-0.34)
β_{ρ}			-1.56				-1.58			-1.58	
			(-2.00)				**(-1.98)**			**(-1.98)**	
β_{ρ}^{Intra}				-0.56		-0.36		-0.32			-0.32
				(-1.60)		(-1.22)		(-1.03)			(-1.04)
β_{ρ}^{Inter}					-1.38	-1.23		-1.27			-1.27
					(-2.05)	**(-1.99)**		**(-2.04)**			**(-2.04)**
β_{int}									-0.10	-0.11	-0.11
									(-0.59)	(-0.65)	(-0.66)
Adj. R^2 (%)	0.09	0.00	0.26	0.10	0.24	0.28	0.36	0.37	0.10	0.37	0.38

Source: UBS Quantitative Research. The table presents the panel regression results from regressing z-scores of the returns of the assets on a list of cross-sectional variables. The t-statistics are calculated using non-parametric Driscoll and Kraay (1998) standard errors that account for heteroskedasticity, cross-sectional and serial correlation. The sample period is from April 1988 to December 2013.

Overall, the results from the alternative specifications do not dramatically change our main finding that changes in correlations and in particular changes in inter-asset-class correlations largely determine the cross-sectional variation of asset returns, even though statistical significance has slightly dropped. The message is still sound and clear.

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