

# Lifecycle Investing in a low interest rate world: Designing a glide path

## *UBS Quantitative Investment Series*

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**ANALYST CERTIFICATION AND REQUIRED DISCLOSURES BEGIN ON SLIDE 33**

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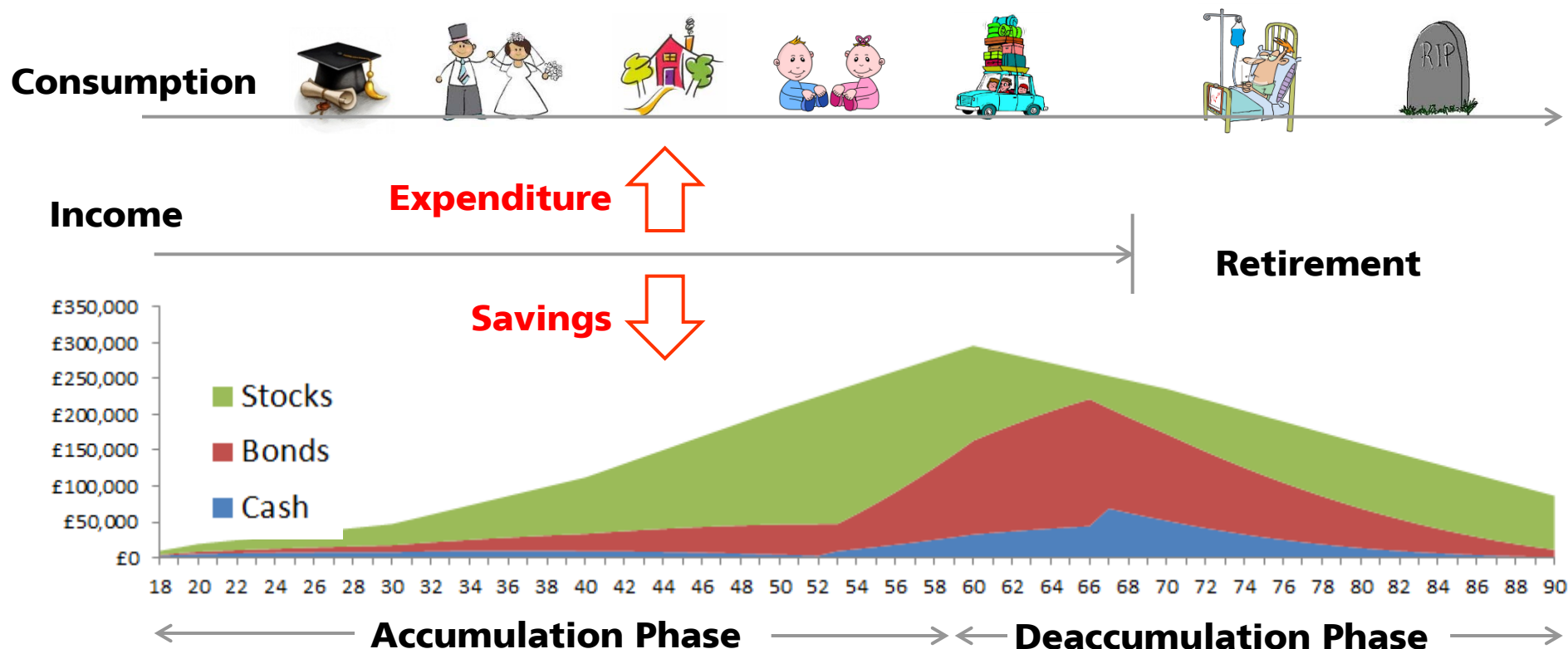
# Outline<sup>1</sup>

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- The Lifecycle Investment Problem
- What the industry does:
  - First wave of products; Target Date Funds
  - Second wave of products; Goal Investing
  - Strategic versus tactical asset allocation
- Solving the life-cycle allocation problem
  - Discussion of approaches
  - A new approach: an explicit solution based on a log-linearisation
- Asset allocation in today's low interest rate environment
  - Set up a multi-asset allocation problem
  - Breaking down the allocation into the non-dynamic, hedging and tactical allocations

1. The author would like to thank Professor James Sefton of Imperial College, London for the significant contribution he provided to this report.

# Lifecycle Investment Problem



- The lifecycle decisions:

1. How much to save and when?

(Savings decision)

2. How to save it – Cash, Bonds or Stocks?

(Portfolio decision)

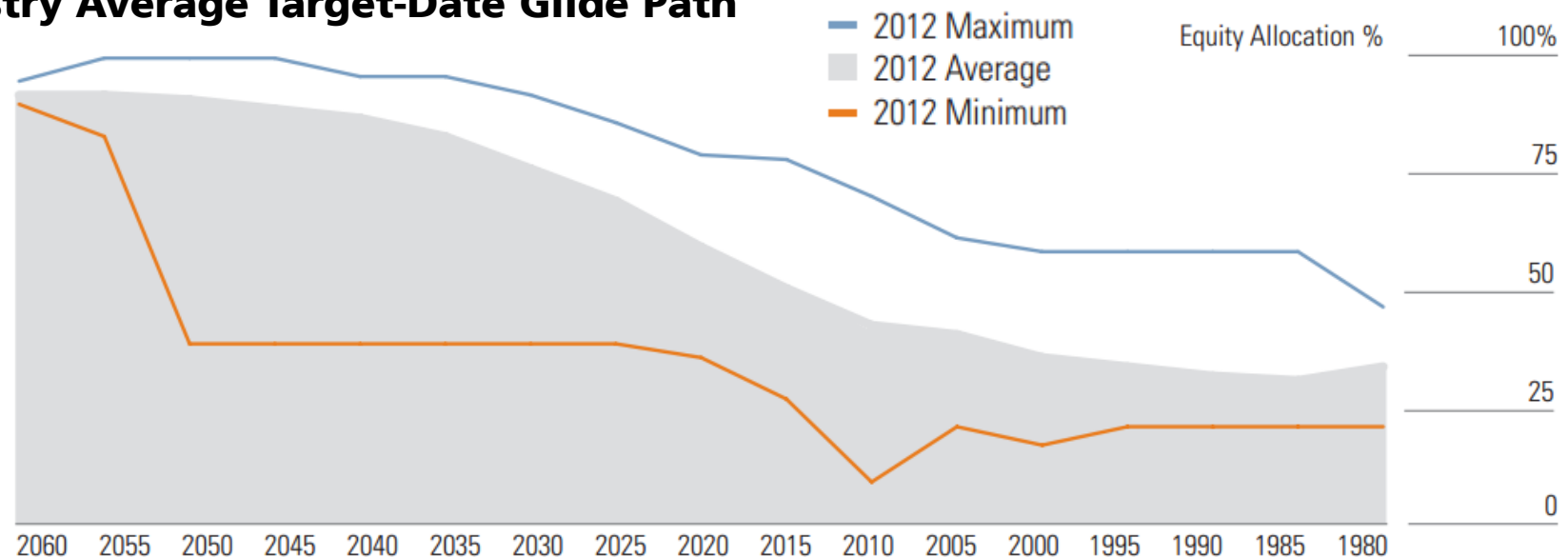
3. When to retire?

(Retirement decision)

# First Wave of Life-Cycle Funds – Target Date Funds (TDF)

- Target Date Funds automatically adjust the Equity / Bond mix over the lifecycle – Glide Path
  - Roughly follow John Bogle's (founder of Vanguard) rule – hold your age as a percentage in bonds (i.e. 100 minus your age in equities).
- Run as a series of funds labelled by the target retirement date, eg, 2015, 2020, ... 2050
  - Investors choose fund corresponding to their target date.
- Assets in TDF topped \$500 billion in the US in early 2013 – 73% of AUM in big three. In the UK, the industry is still young and assets in the principal TDF providers were just over £5 billion in 2013.

## US Industry Average Target-Date Glide Path



Source: Exhibit 6. 'Target-Date Series Research Paper 2013 Survey', © 2013 Morningstar, Inc. All Rights Reserved. Reproduced with permission. Data as of 12/31/2012 (Please see note on p27).

# Second Wave of Life-Cycle Funds – Goal Investing

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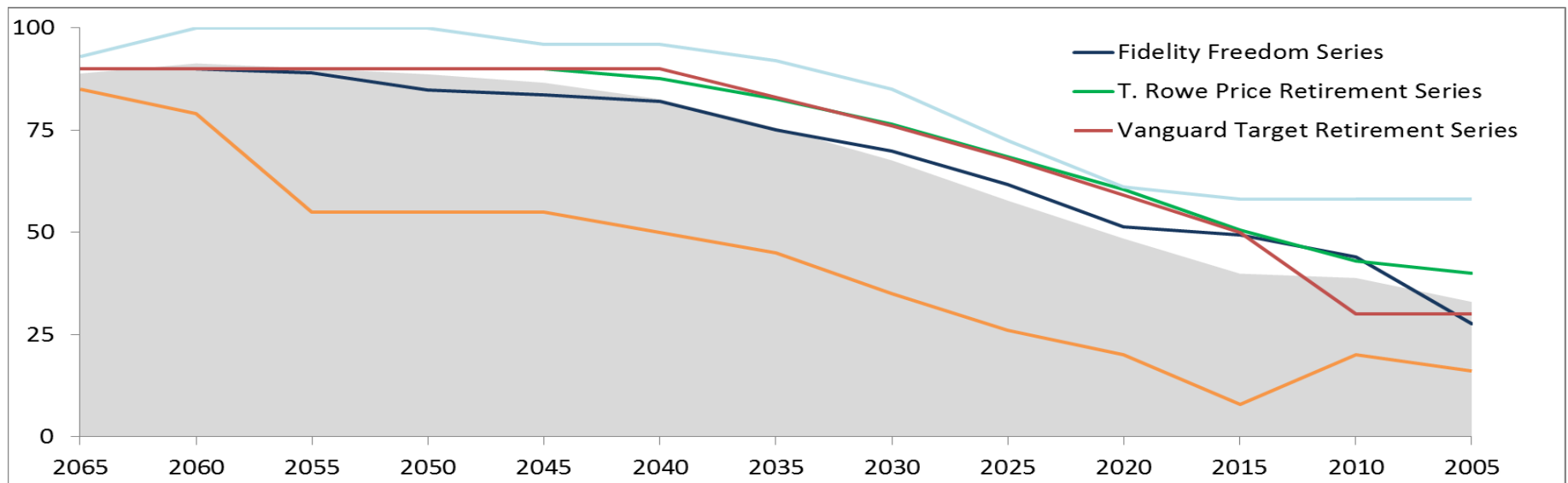
- Pioneered by Robert Merton, Goal-Based Investing focuses on both a minimum and a desired level of inflation-protected income in retirement.
  - The spread in the two income levels implicitly determines investor risk aversion.
- Given the contribution rate, the investment strategy is designed to maximise the probability of achieving the desired income subject to making the minimum level with 95% probability.
  - If the probability of reaching the desired income level is too low, then the contribution rate must be increased.
  - Asset allocation strategy is dynamic but hidden. Based on three funds:
    - World Equity Fund
    - Long-Term Index-Linked Bond Portfolio
    - Short-Term Index-Linked Bond Portfolio

Information in this slide is based on Robert Merton's presentation to the NEST Forum 2013, June 2013, London and Wade Pfau (2013)

# Strategic and Tactical Asset Allocation

- Strategic Asset Allocation problem is the design of the glide path – in the diagram the percentage allocation to equity but more generally the domestic vs foreign equity, bonds and cash allocation.
- Most funds allow for some tactical allocation around the glide path based on current conditions.
  - August 2014, Fidelity announced that they will increase the tactical allocation around their glide path to +/- 10%.
  - T. Rowe Price limit their tactical allocation to +/- 5% around the glide path.
  - Vanguard do not tactically allocate.

## Equity Allocation of the Largest three by AUM



Source: 'Target-Date Series Research Paper 2013 Survey', © 2013 Morningstar, Inc. All Rights Reserved. Reproduced with permission. Data as of 12/31/2012 (Please see note on p27).

# Design of Strategic and Tactical Asset Allocation Strategy

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- There are a number of quantitative approaches that can be used to inform the design of the glide path and any tactical allocation.
- It is inherently a difficult problem even in the simplest cases:
  - Necessary to solve over the whole life path simultaneously – my decision today depends on my decision tomorrow
  - It depends on the probability distribution of outcomes – so my decision tomorrow depends on the outcome today
  - With a lot of variables and choices, the problem soon becomes too big to solve – ‘the curse of dimensionality’
- There are number of approaches to investigating this problem:
  1. Monte-Carlo Simulation approach
  2. Exact Numerical Solution of the non-linear models using grid methods
  3. Explicit solutions of linearised models

# Design of Strategic and Tactical Asset Allocation Strategy

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## 1. Monte-Carlo Simulation approach.

- Detemple, Garcia et al (2003), Brandt et al (2005), Basu and Drew (2009)

Advantage	Disadvantage
<ol style="list-style-type: none"><li>1. Copes with high dimensional problems</li><li>2. Can bootstrap returns to capture tail events</li><li>3. Good for performance testing</li></ol>	<ol style="list-style-type: none"><li>1. Hard to optimise – requires a parameterization of policy function.</li><li>2. Need to search over all parameters of the parameterization</li></ol>

## 2. Exact Numerical Solution of the non-linear models using grid methods

1. Cocco et al, (2005). Gomes and Michaelides (2005)

<ol style="list-style-type: none"><li>1. Solves for the optimal policy</li><li>2. General - can cope with constraints, non linearities, etc</li></ol>	<ol style="list-style-type: none"><li>1. Computationally intensive.</li><li>2. Soon suffers from curse of dimensionality</li></ol>
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## 3. Explicit solutions of linearised models

1. Jurek and Viceira (2010), Campbell, Chan, Viceira (2003), Collin-Dufresne et al. (2012).

<ol style="list-style-type: none"><li>1. Solves for the optimal policy function</li><li>2. Can cope with high dimensions</li><li>3. Intuitive</li></ol>	<ol style="list-style-type: none"><li>1. Cannot cope with inequality constraints or non-linearities</li></ol>
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# Model

- In order to maximise welfare given by

$$E_t \left( \frac{1}{1-\beta} \sum_{i=0}^T C_{t+i}^{1-\beta} (1-\delta)^i \right)$$

Coefficient of risk aversion

$t$  denotes time,  $i$  denotes age

- Households choose both consumption and their portfolio weights,  $n$ , where

$$R_{t+1}^p = n^T (R_{t+1} - R_{t+1}^0) + R_{t+1}^0 \quad \text{where} \quad \log \begin{bmatrix} R_{t+1}^0 \\ R_{t+1} - R_{t+1}^0 \end{bmatrix} \rightarrow N \left( \begin{bmatrix} \mu_{t+1}^0 \\ \mu_{t+1}^r \end{bmatrix}, \begin{bmatrix} \Sigma_{00} & \Sigma_{0r} \\ \Sigma_{0r}^T & \Sigma_{rr} \end{bmatrix} \right)$$

- Subject to the budget constraint

$$W_{t+1} = R_{t+1}^p (W_t + Y_t - C_t)$$

where  $W_t = W_0$

and  $W_t > 0$  for all  $t$  (No Borrowing)

and  $Y_{t+i} = 0$  if  $i > t_{ret}$  (Retirement)

Time-varying returns

Retirement Fixed

# Log-Linearisation

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- Use the log-linearisation approach of Campbell, Shiller. Denote the log of the various variable by their lower case - i.e.  $c = \log C$  etc.
- Then the log-linear wealth identity becomes

$$w_{t+1} = r_{t+1}^p + \rho_i^y (y_t - w_{t+i}) - \rho_i^c (c_t - w_{t+i}) + w_t + \kappa_i$$

 This term disappears in retirement

where we have linearised around the expected log of consumption/wealth ratio and income/wealth ratios at age  $i$

Important: the coefficients  $\rho^y$ ,  $\rho^c$  and  $\kappa$  vary with the age,  $i$ , of the household not time.

- Similarly the portfolio returns can be written

$$r_{t+1}^p = n^T (r_{t+1} - r_{t+1}^0) + \frac{1}{2} n^T (\sigma^2 - \Sigma n) \quad \text{where } \sigma^2 = \text{diag}(\Sigma)$$

# Time-Varying Opportunity Set

- The time variation of returns and income can be described by an AR(1) process  
(Can make it as involved as you like as long as it is linear!)
- The states of the AR process are

$$\underbrace{\begin{bmatrix} r_{t+1}^0 \\ r_{t+1} \\ s_{t+1} \\ y_{t+1} \end{bmatrix}}_{x_{t+1}} = A \underbrace{\begin{bmatrix} r_t^0 \\ r_t \\ s_t \\ y_t \end{bmatrix}}_{x_t} + \underbrace{\begin{bmatrix} \alpha^0 \\ \alpha^r \\ \alpha^s \\ \alpha^y \end{bmatrix}}_{\alpha} + B \underbrace{\begin{bmatrix} \varepsilon_{t+1}^0 \\ \varepsilon_{t+1}^r \\ \varepsilon_{t+1}^s \\ \varepsilon_{t+1}^y \end{bmatrix}}_{\varepsilon_{t+1}} \quad \text{where} \quad \text{Var}(\varepsilon_{t+1}) = \underbrace{\begin{bmatrix} \Sigma_{00} & \Sigma_{0r} & \Sigma_{0s} & \Sigma_{0y} \\ \cdot & \Sigma_{rr} & \Sigma_{rs} & \Sigma_{ry} \\ \cdot & \cdot & \Sigma_{ss} & \Sigma_{sy} \\ \cdot & \cdot & \cdot & \Sigma_{yy} \end{bmatrix}}_{\Sigma}$$

- It will be useful later to write  $r_t = C_1 x_t$  and  $r_t^0 = C_0 x_t$
- The conditioning states,  $s_t$ , are variables such as the dividend/price ratio. The conditional means of the assets is then  $\mu_t = C_1 (A x_t + \alpha)$
- Income process can depend on conditioning variables as well. This induces correlations between income and states.

# Income Process

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- Two states are needed to describe the household income process.
- Assume household income is equal to the average aggregate real wage,  $WG$ , times idiosyncratic household productivity,  $Y^{id}$ .

$$Y_t = RW_t \times Y_t^{Id} \quad \Rightarrow$$

$$y_t = rw_t + y_t^{Id}$$

- The aggregate wage is just the sum of real wage growth (one of our conditioning variables)

$$rw_{t+1} = rw_t + gRW_{t+1}$$

- And we assume idiosyncratic productivity evolves as

$$y_{t+1}^{Id} = 0.99y_t^{Id} + 0.01\alpha^{Id} + \varepsilon_{t+1}^{Id}$$

- In our simulation we shall assume a long run productivity of  $\alpha^{Id} = \log(100,000/4)$  and initialise our household with an income of  $y_o^{Id} = \log(75,000/4)$ . The innovations are assumed to uncorrelated with the innovations to other state variables and have an annualised volatility of 5%.

# Solution Procedure (1)

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- The Value function of the household at age  $i$  – maximal achievable welfare – is a quadratic function of the states and wealth

$$J_t = -k_i e^{-\gamma_i \left( w_t + \frac{1}{2} x_t^T \Pi_i x_t - x_t^T \Phi_i + \phi_i \right)}$$

- We can solve the Bellman equation

$$J_t = \max_{c_t, n_t} \left( \frac{1}{(1-\beta)} e^{(1-\beta)c_t} + E_t (J_{t+1}) \right)$$

- Using the following powerful result from Whittle (1981), Lemma 6.1.2 – which is the basis of risk-sensitive control and the robustness results of Hansen and Sargent. Assume  $Q(n, x, \varepsilon)$  is quadratic in its variables then

$$\max_n \int -e^{-\frac{1}{2} Q(n, x, \varepsilon)} d\varepsilon = -k e^{-\frac{1}{2} \max_n \min_{\varepsilon} Q(n, x, \varepsilon)}$$

Worst case  
perturbation

## Solution Procedure (2)

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- Initialize the parameters  $\rho^y$ ,  $\rho^c$  and  $\kappa$  and set  $\Pi_T = \Phi_T = \varphi_T = 0$
- Use Whittle's Lemma to solve the maximisation problem at T-1 then T-2 etc. Gives an backward iteration formula – the famous risk-sensitive Riccati equations – for  $\Pi_i$ ,  $\Phi_i$ ,  $\varphi_i$  for  $i=T-1, T-2, T-3$ , etc. Similarly iterate backwards for the parameters  $\gamma_i$  and  $k_i$
- We can express the optimal portfolio and consumption in terms of these parameters

Let 
$$V_{i+1} = \begin{bmatrix} B^T \Pi_{i+1} B + \frac{1}{\gamma_{i+1}} \Sigma^{-1} & B^T C_1^T \\ C_1 B & -C_1 B \Sigma B^T C_1^T \end{bmatrix}$$
 then

Worst case perturbation  $\rightarrow$  
$$\begin{bmatrix} \varepsilon_{t+i}^* \\ n_{t+i} \end{bmatrix} = -V_{i+1}^{-1} \left( \begin{bmatrix} B^T \Pi_{i+1} A x_{t+i} - B^T (\Phi_{i+1} - \Pi_{i+1} \alpha - C_0^T) \\ C_1 (A x_{t+i} + \alpha) \end{bmatrix} \right)$$
 Hedging Portfolios

$$c_{t+i} = \frac{1}{1-\beta} \log \left( \frac{\gamma_i \rho_i^c}{(1 - \rho_i^y + \rho_i^c)} J_{t+i} \right)$$
 Conditional Mean Return

- Having computed a path for all our variables, we can re-estimate the linearisation parameters  $\rho^y$ ,  $\rho^c$  and  $\kappa$ . We repeat until convergence.

# The Data Set

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- We now describe how our world varies over time. Let the assets be:

$r_0$		Real Return to 3M Treasury Bills
r	xRe	Excess returns to S&P500
	xRb	Excess returns to 10yr Government Bond Index

- Let the conditioning states be

s	NY	Nominal 1 year Yield
	SPR	Term Spread - 10 year minus 1 year nominal yields
	DP	Dividend – Price ratio on the S&P 500
	gRGDP	Growth rate in real US GDP (NIPA)
	gRw	Growth rate in real wage compensation (NIPA)

- We estimate our process on quarterly data from 1971Q4 – 2014Q2.

# The Estimated VAR

- The table below records the estimates of the AR(1) coefficients
  - Coefficients significant at the 95% confidence level are in bold
- The persistent state variables forecast returns to cash, equity and bonds.
- Returns to the indexed linked bonds and the long-short value growth portfolio are only mildly forecastable.

Time varying forecastable asset returns

	Constant (Long Run)	Real 3M Returns	Excess S&P500	Excess 10 yr Bond	Nominal 3M Yield	Term Spread	Dividend- Price Ratio	GDP Growth	Real Wage Growth
Real 3M Returns	1.04%	<b>0.33</b>	0.00	0.02	<b>0.22</b>	<b>0.25</b>	<b>-0.31</b>	0.04	-0.10
Excess S&P500	5.36%	2.06	0.11	0.11	<b>-1.51</b>	-1.21	<b>2.94</b>	1.68	-0.79
Excess 10 yr Bond	2.42%	0.06	<b>-0.09</b>	-0.04	<b>0.57</b>	<b>1.65</b>	<b>-1.11</b>	0.03	-0.09
Nominal 3M Yield	5.12%	-0.22	0.02	0.02	<b>0.96</b>	-0.01	0.07	-0.08	<b>0.22</b>
Term Spread	1.68%	0.26	0.00	-0.02	-0.07	<b>0.74</b>	0.057	0.05	<b>-0.22</b>
Dividend-Price Ratio	3.11%	-0.05	0.00	-0.01	0.02	-0.03	<b>0.92</b>	-0.06	0.04
GDP Growth	2.83%	-0.20	<b>0.02</b>	0.01	0.08	<b>0.15</b>	-0.10	<b>0.19</b>	<b>0.22</b>
Real Wage Growth	1.85%	-0.06	<b>0.02</b>	0.01	<b>0.11</b>	<b>0.22</b>	<b>-0.26</b>	<b>0.52</b>	-0.08

Persistent Conditioning Variables

Source: UBS Quants



# The Covariance Matrix

- The table displays information on the estimated covariance matrix.
  - The standard deviations are recorded on the main diagonal and the correlations on the off diagonal.
- The first column records the sample volatility –annualised for all asset returns and macro growth variables.

	Sample Volatility	Real 3M Returns	Excess S&P500	Excess 10 yr Bond	Nominal 3M Yield	Term Spread	Dividend-Price Ratio	GDP Growth	Real Wage Growth
Real 3M Returns	1.53%	0.54%	0.11	0.23	-0.06	-0.05	-0.06	-0.09	<b>0.48</b>
Excess S&P500	16.78%		8.03%	0.02	-0.05	0.03	<b>-0.69</b>	0.14	0.06
Excess 10 yr Bond	9.33%			4.40%	<b>-0.58</b>	-0.06	-0.11	<b>-0.37</b>	0.08
Nominal 3M Yield	2.99%				0.92%	<b>-0.62</b>	0.29	0.27	0.06
Term Spread	1.18%					0.68%	0.12	-0.02	-0.09
Dividend-Price Ratio	1.17%						0.35%	-0.06	0.03
GDP Growth	1.64%							0.71%	0.29
Real Wage Growth	2.03%								0.84%

Will drive strong hedging motive

Source: UBS Quants

# Identifying Restrictions (Recursive Structure)

- To identify shocks assume:
  - GDP shocks are exogenous shocks to productivity.
  - Inflation shocks are exogenous shocks uncorrelated with productivity shocks.
  - Monetary Shocks are exogenous shocks to nominal yields uncorrelated with either productivity and inflation shocks.
  - Equity valuation shocks are exogenous shocks to D/P ratio uncorrelated with the above.

	Productivity Shock	Inflation Shock	Monetary Shocks	Equity Valuation Shock
Real 3M Returns	0.07%	-0.36%	0.02%	0.14%
Excess S&P500	-1.57%	-0.25%	0.82%	16.84%
Excess 10 yr Bond	2.28%	-1.08%	2.48%	-0.30%
Nominal 3M Yield	-0.34%	0.02%	<b>-1.00%</b>	0%
Term Spread	0.02%	0.08%	0.49%	-0.72%
Dividend-Price Ratio	0.03%	-0.02%	-0.13%	<b>-1.00%</b>
GDP Growth	<b>-1.00%</b>	0%	0%	0%
Real Wage Growth	-0.34%	<b>-1.00%</b>	0%	0%

Bonds provide a hedge to productivity but not inflation shocks

Source: UBS Quants

Equity does not provide a hedge to either

# Model Parameters

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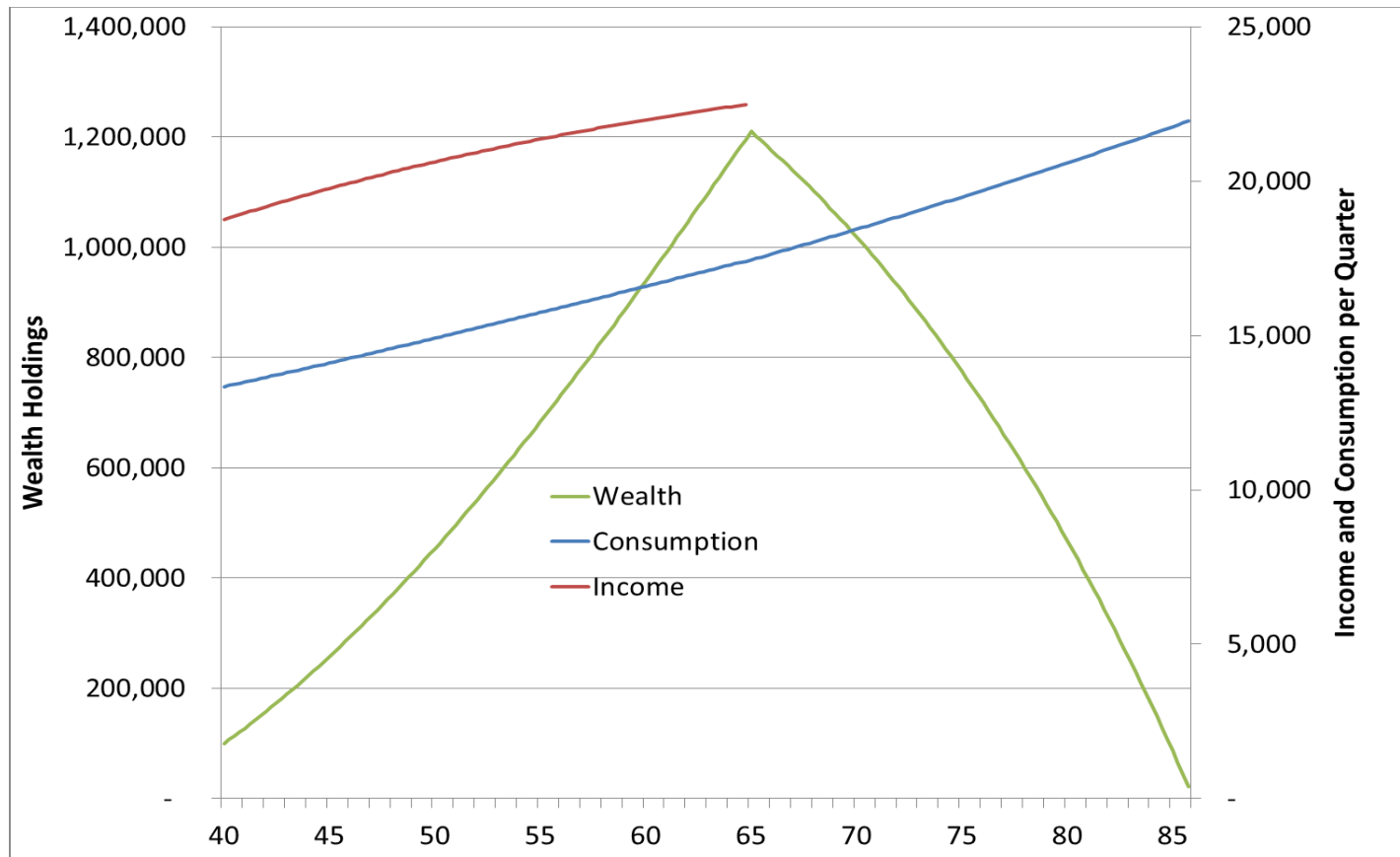
- Assume our household is aged 40 and has 100,000 wealth
- We assume a  $\delta = 0.5\%$  per year and risk aversion,  $\beta$ , of 7.5.
- As with static portfolio problems it is necessary to shrink the long run mean returns. We set

Expected annualised long run returns to assets	Lon Run	2014Q2	2009 Q1
Real Return to 3M Treasury Bills	1.5%	0.0%	-1%
Excess returns to S&P500	4%	11.1%	12.2%
Excess returns to 10yr Government Bond Index	1.0%	1.3%	-0.1%
Nominal Yield (1 Year)	3.5%	1.1%	1.1%
Term Spread (10 year minus 1 Year)	1.5%	2.1%	2.5%
Dividend Price Ratio	2.5%	2.1%	3.1%
GDP Growth	2.8%	3.5%	0.9%
Real wage Growth	1.6%	2.7%	-1.6%

- This affects the mean-variance portfolio or long run position. It has no effect on either the hedging portfolios or the tactical allocation.

# Cash, Bond and Equity Allocation problem

- We solve the reduced problem first – constraining the household to hold only cash bonds or the equity portfolio.
- We observe the normal life-cycle savings plan
  - Saves roughly 20% of income to target wealth holdings of 1,200,000 at retirement

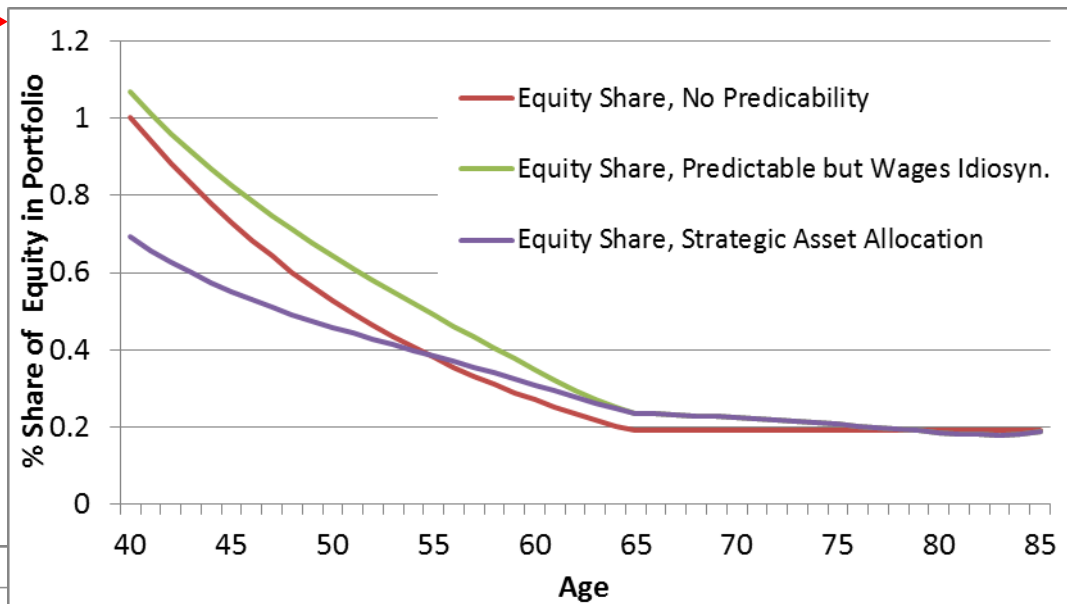


Source: UBS Quants

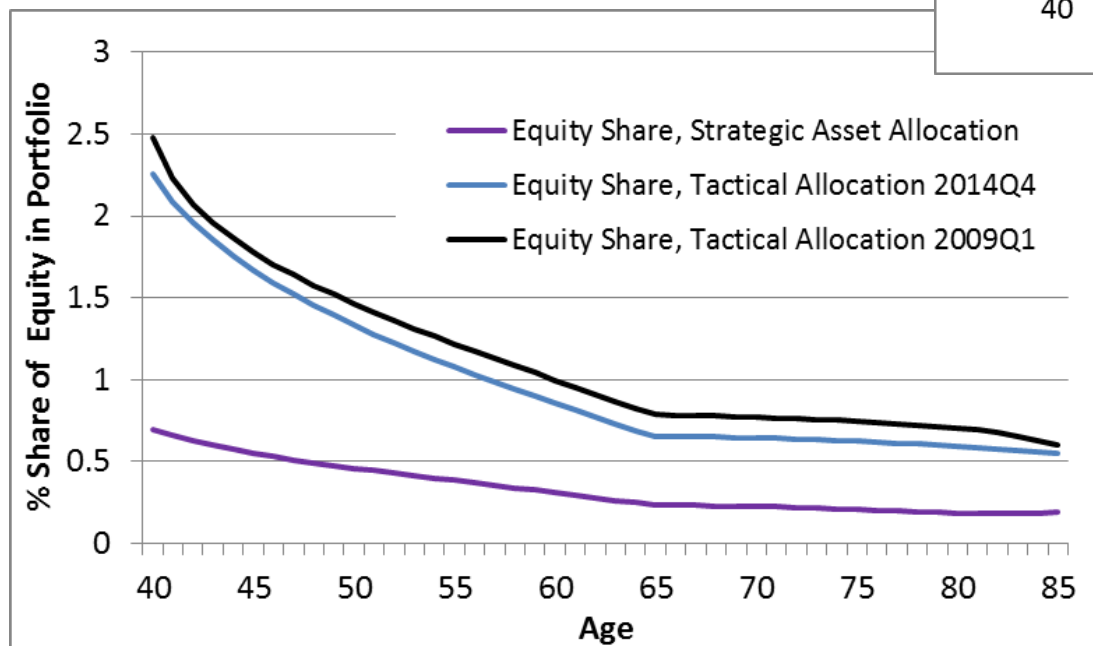
# Cash and Equity Portfolio only, Moderate Risk Aversion $\beta=7.5$

- Strategic Asset Allocation

1. No predictability in asset returns ( $A=0$ )
2. Predictable Returns but wages idiosyncratic
3. Full Model – but states at long run equilibrium values (Strategic Allocation)



Source: UBS Quants



Source: UBS Quants

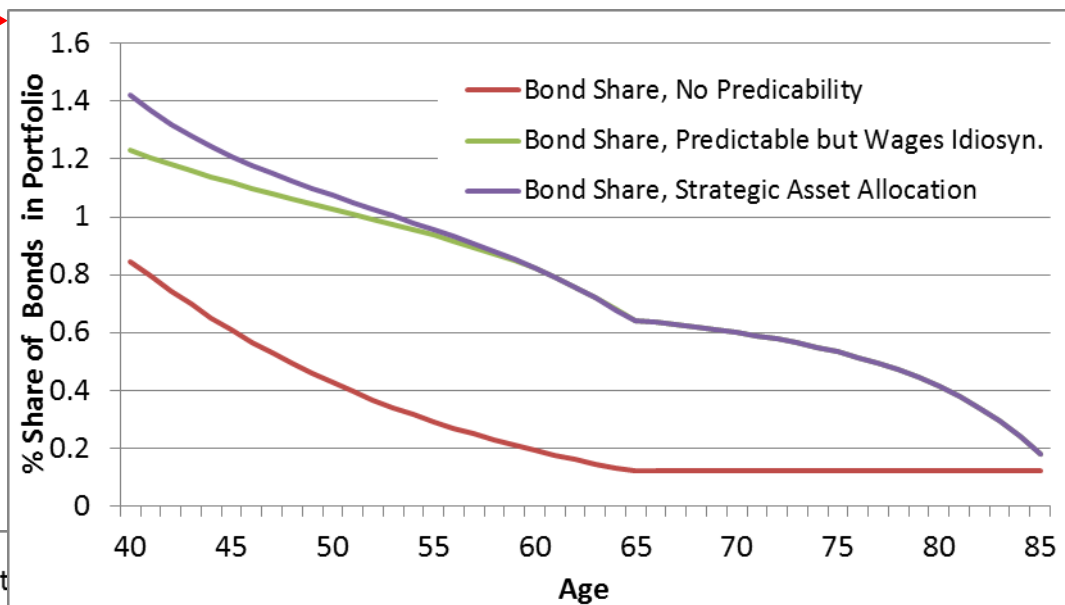
- Tactical Asset Allocation

1. Set the states at their value in 2014Q2 (Inflation lower, GDP recovered)
2. Set the states at their value in 2009Q1 (Inflation high so real yields lower, GDP growth v. low)

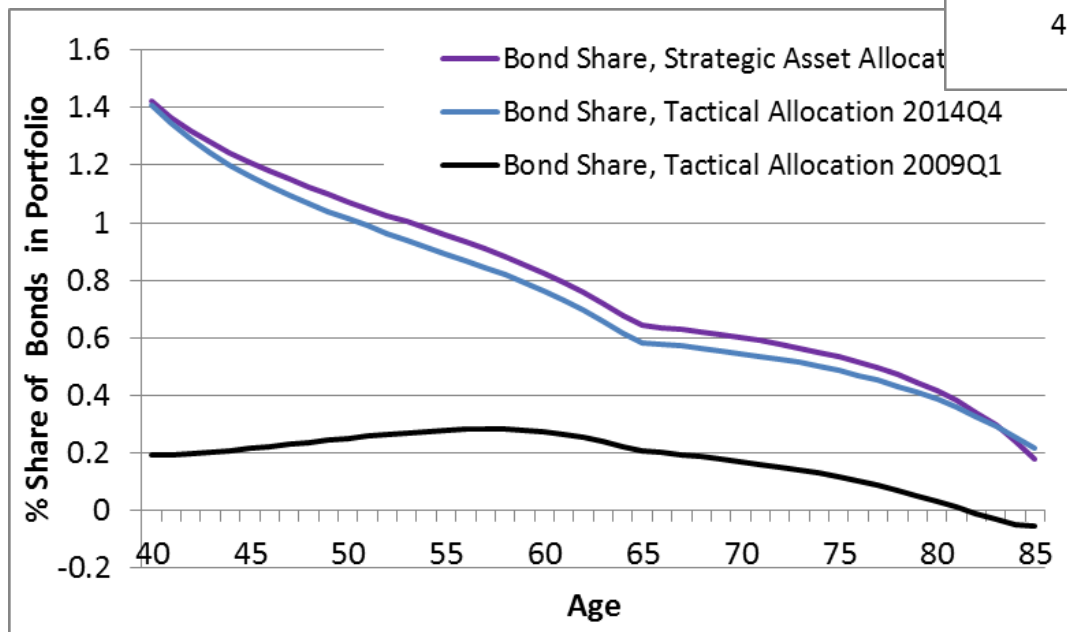
# Cash and Bond Portfolio only, Moderate Risk Aversion $\beta=7.5$

- Strategic Asset Allocation

1. No predictability in asset returns ( $A=0$ )
2. Predictable Returns but wages idiosyncratic
3. Full Model – but states at long run equilibrium values (Strategic Allocation)



Source: UBS Quants



Source: UBS Quants

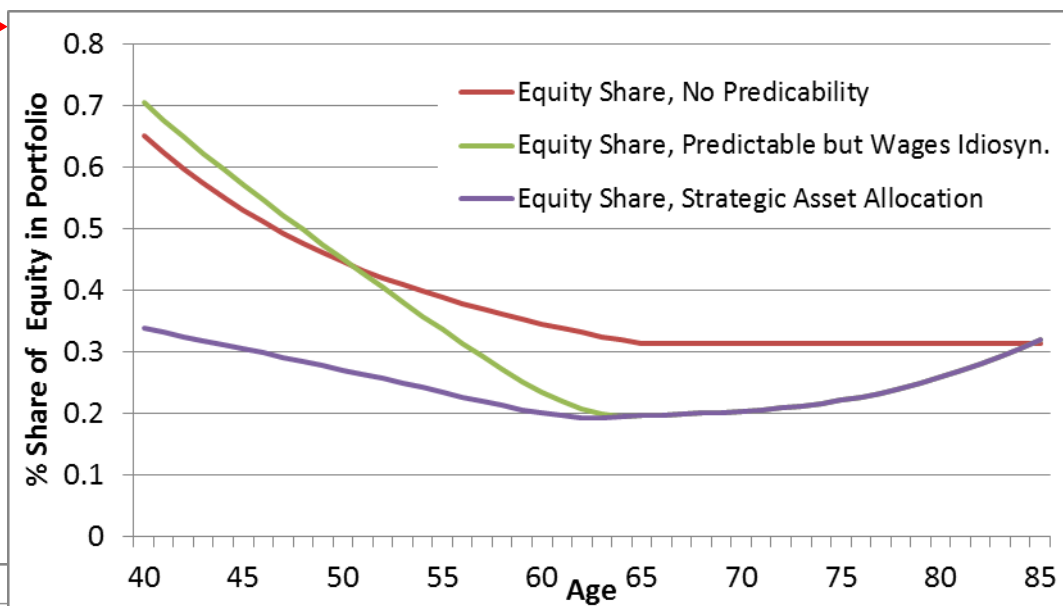
- Tactical Asset Allocation

1. Set the states at their value in 2014Q2 (Inflation lower, GDP recovered)
2. Set the states at their value in 2009Q1 (Inflation high so real yields lower, GDP growth v. low)

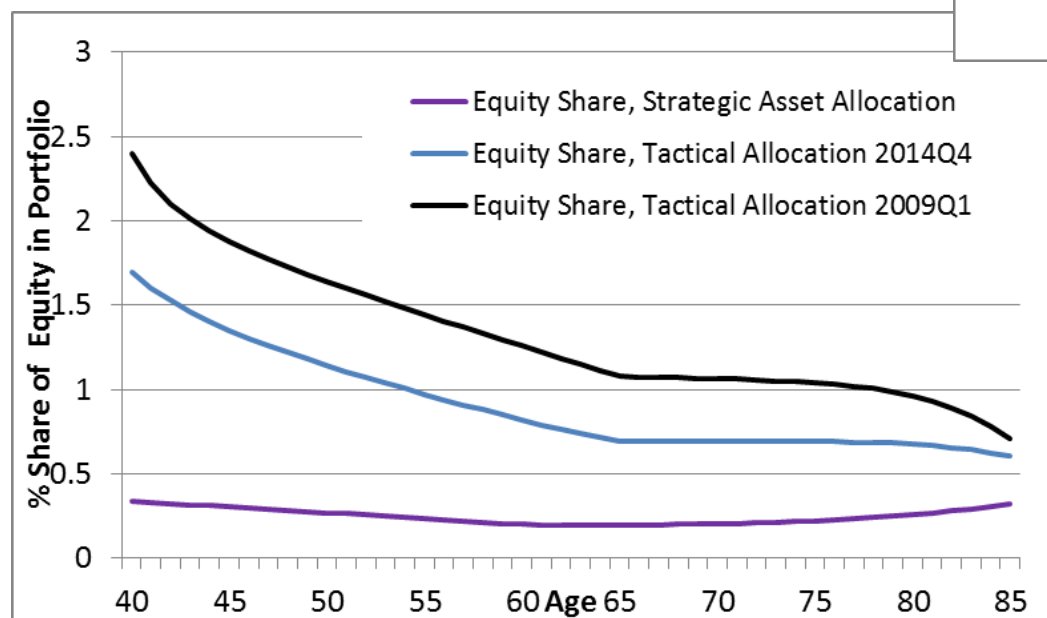
# Bonds and Equity Portfolio only, Moderate Risk Aversion $\beta=7.5$

- Strategic Asset Allocation

1. No predictability in asset returns ( $A=0$ )
2. Predictable Returns but wages idiosyncratic
3. Full Model – but states at long run equilibrium values (Strategic Allocation)



Source: UBS Quants



Source: UBS Quants

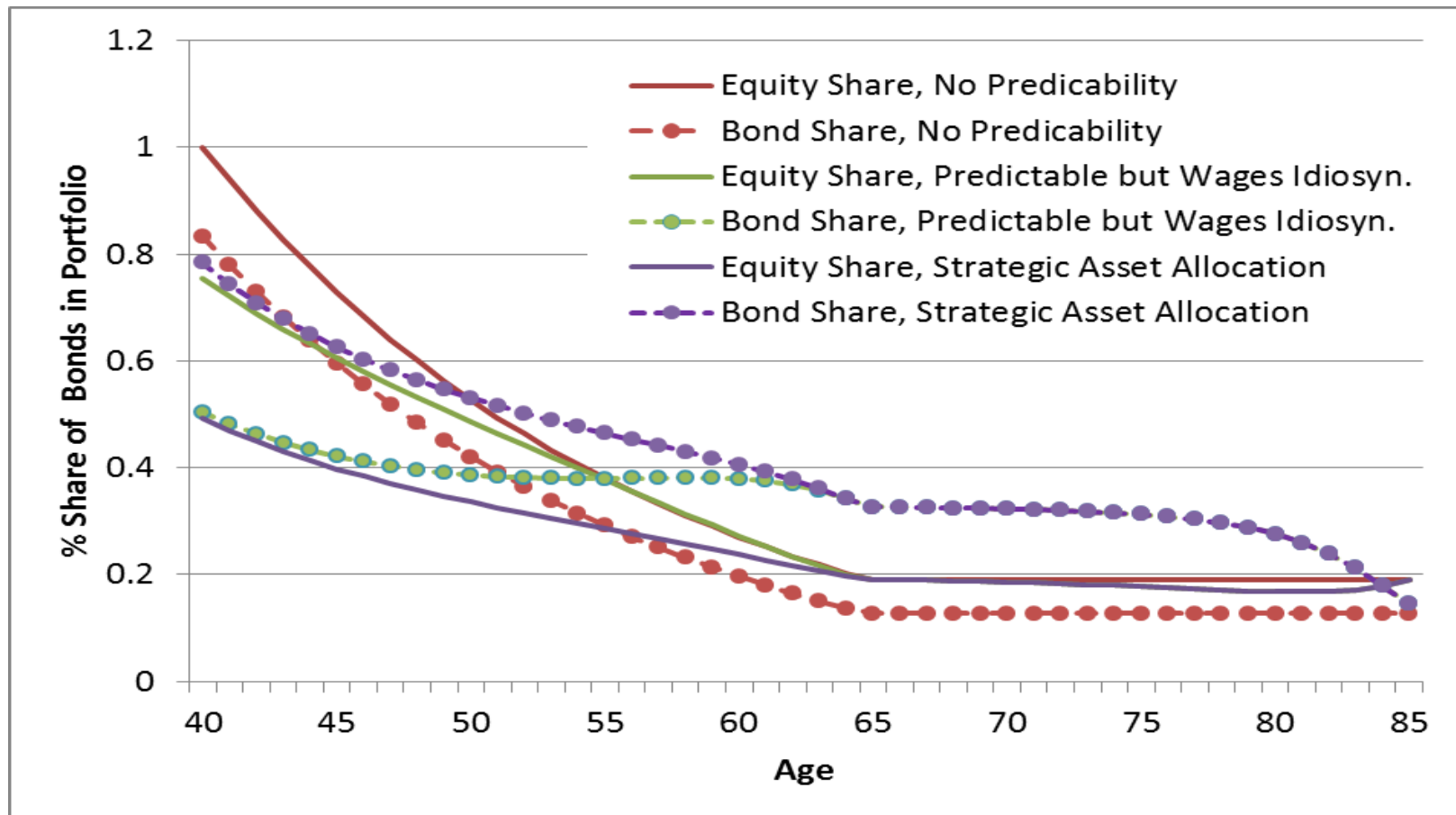
- Tactical Asset Allocation

1. Set the states at their value in 2014Q2 (Inflation lower, GDP recovered)
2. Set the states at their value in 2009Q1 (Inflation high so real yields lower, GDP growth v. low)

# Cash, Bonds and Equity Portfolio only

- In the Strategic Asset Allocation

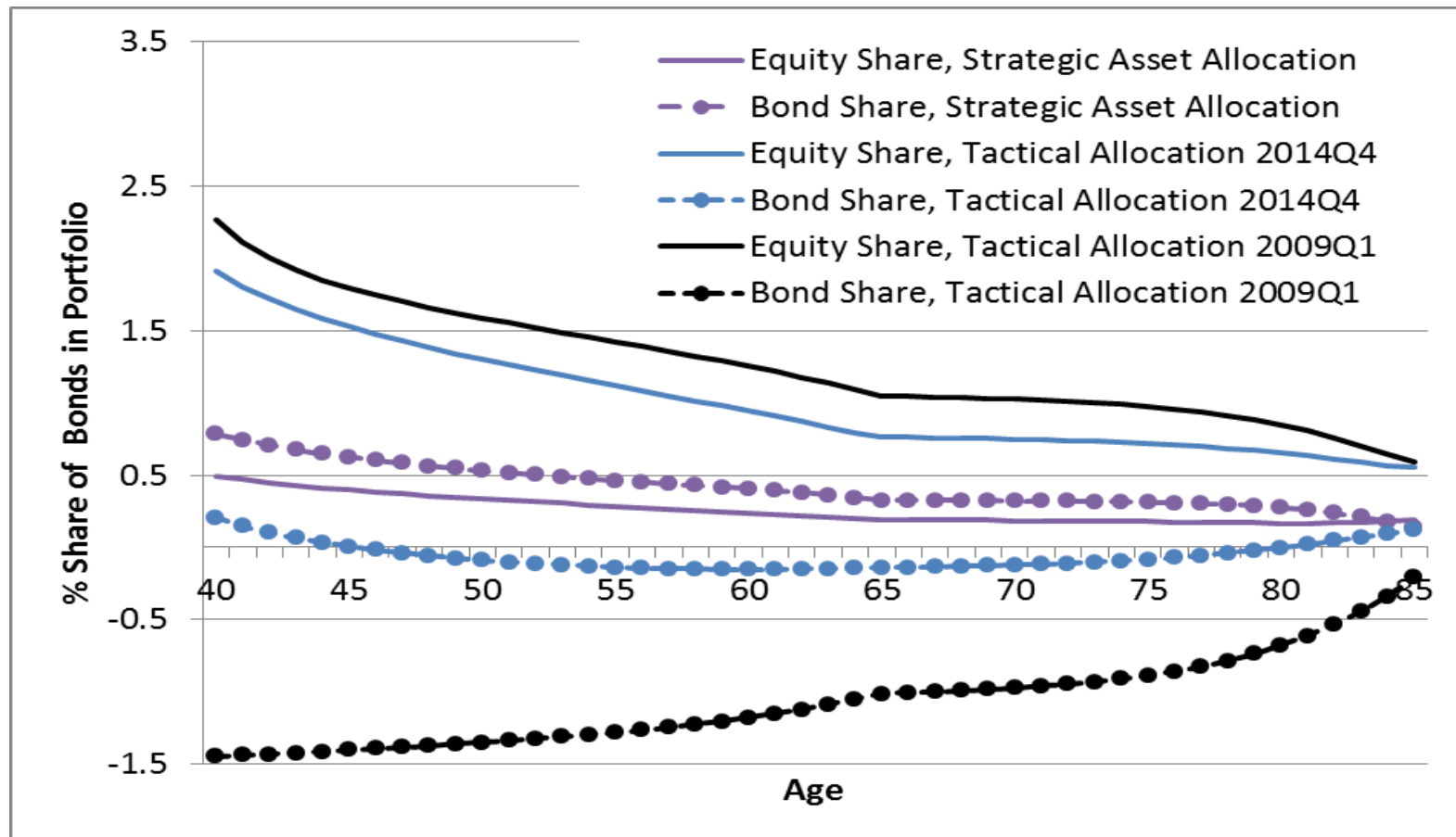
1. Equity allocations are reduced heavily in early life due to joint correlation of equity returns and real wages to productivity shocks
2. Equity allocations are reduced in later life due to strong motive to hedge bond returns





# Cash, Bonds and Equity Portfolio only

- In the Tactical Asset Allocation
  1. Equity allocations are increased heavily due to expected high returns.
  2. Since 2009Q1 the allocations have been reduced but still very significant



# Introduce Real Consols

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- We introduce synthetic real consols into the asset allocation problem.
- N.B. Experimented with data on returns to indexed linked bonds – did not work.
  1. Short time series - only reliable in US since 2003
  2. Very highly correlated with nominal bonds (low inflation environment).
- The returns to our synthetic real bonds are calculated using the log-linear formula of Campbell, Lo and MacKinlay (1997, p408) assuming no risk premium.

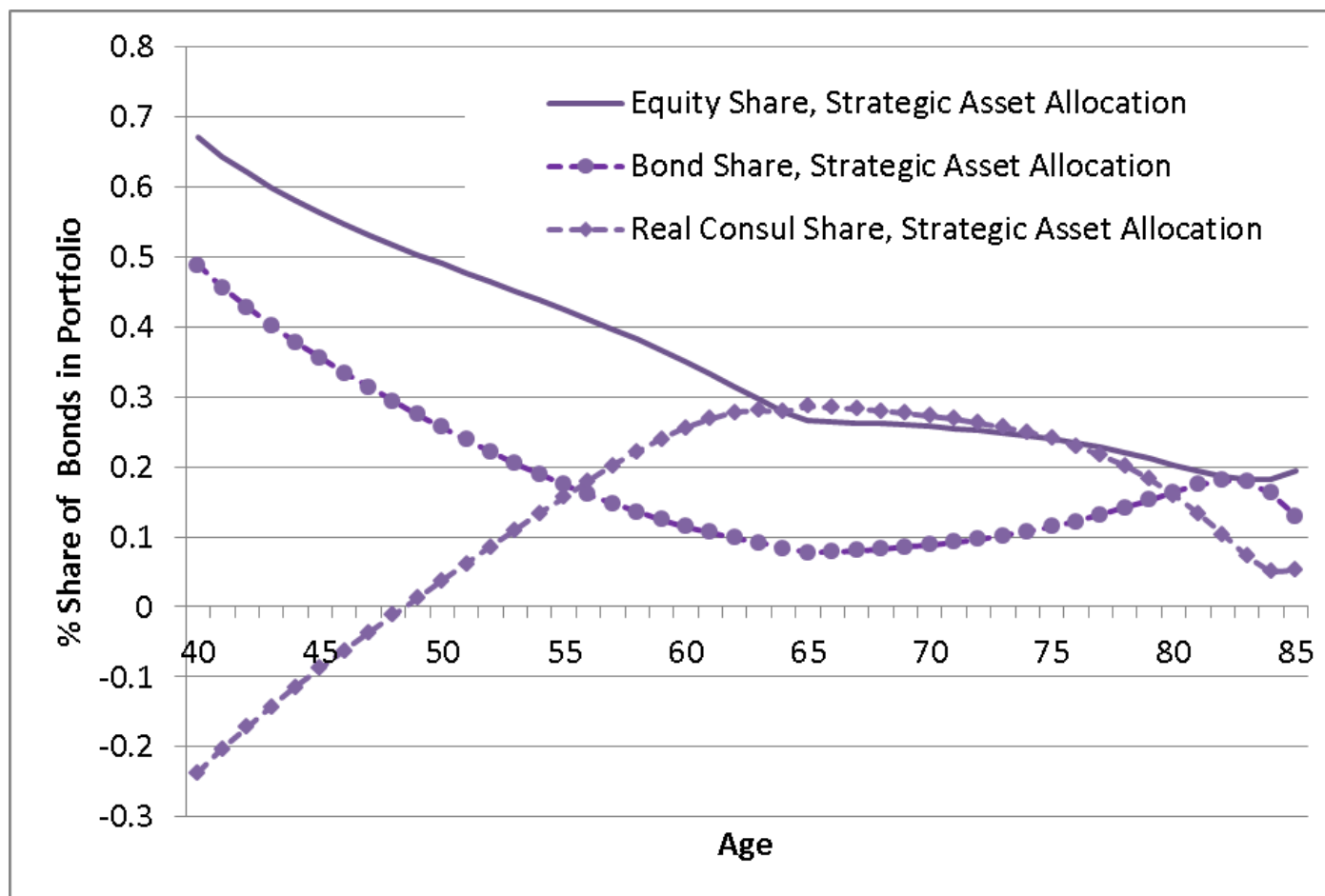
$$r_{t+1}^{consol} = E_t(r_{0,t+1}) - (E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho^j r_{0,t+1+j}$$

where  $\rho = \frac{1}{1 + E(r_{0,t+1})}$  the unconditional expected cum-dividend price of the consol.

- Given our VAR formulation for returns, the returns to the synthetic real consol can be expressed as a function of the transition matrix A.

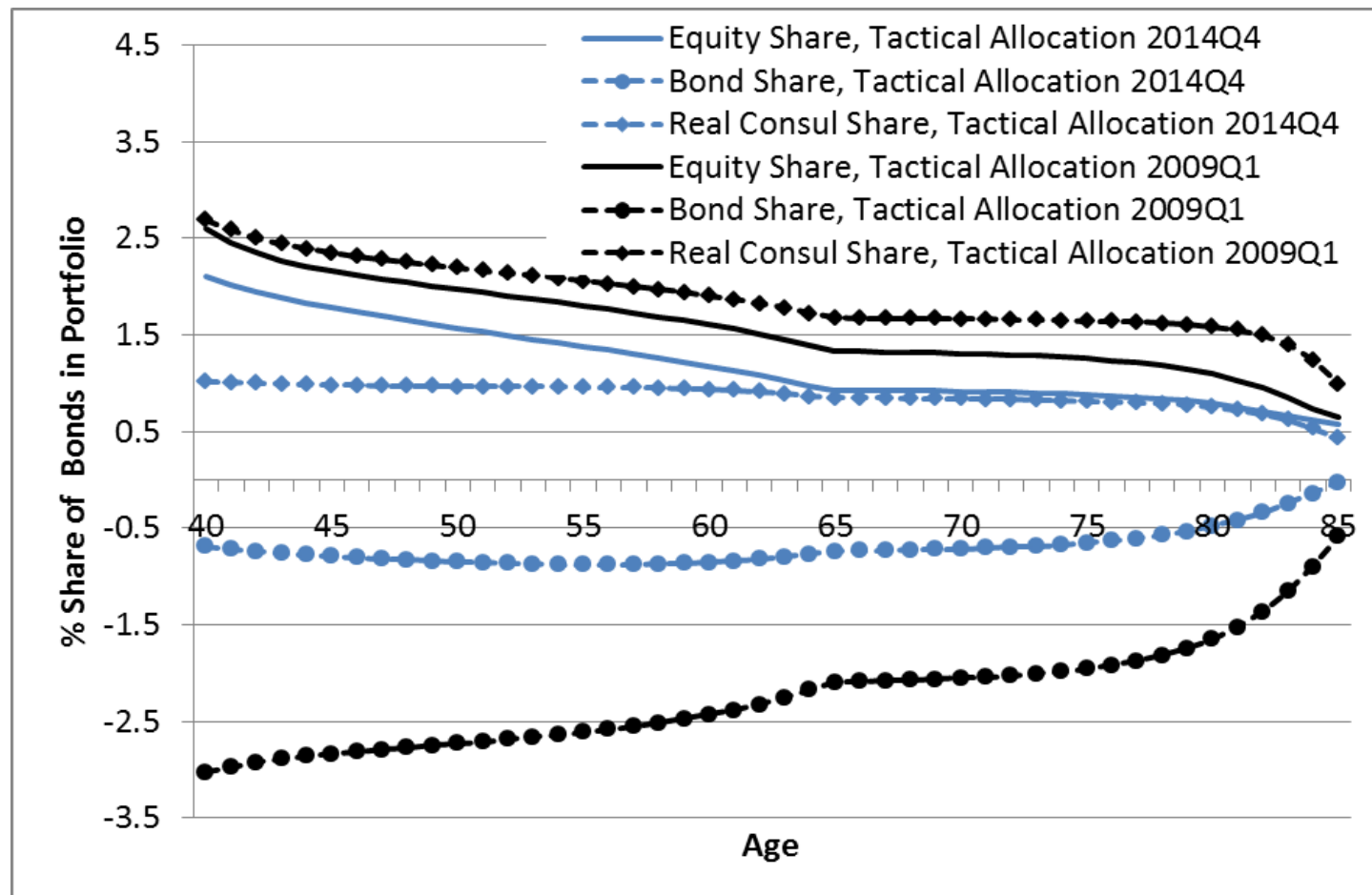
# Cash, Consols, Bonds and Equity Portfolio

- In the Strategic Asset Allocation
  1. The equity glide path looks very similar to current wisdom.
  2. However as investor ages, she shifts into real consols and out of bonds.



# Cash, Consols, Bonds and Equity Portfolio

- In the Tactical Asset Allocation
  1. Allocations are more extreme as the investor can now hedge the real wage risk more effectively.



# Conclusions

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- Developed a new approach to solving the life-cycle portfolio allocation problem:
  - Uses the log-linear approach of Campbell and Shiller
  - Close in spirit to the Jurek and Viceira (2010) paper adapted to the life-cycle problem
- Approach can be generalised to include:
  - State pension income in retirement
  - Annuitisation
  - Uncertain time of death
  - Bequest Motive
  - Non-Expected Utility function to separate out the risk aversion from the elasticity of substitution
- Applied approach to tactical asset allocation in a low interest environment

# Appendix: The Riccati equations (1)

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- Define the additional parameter  $\rho_i^w = 1 - \rho_i^y + \rho_i^c$
- Then the backward iteration for the constants are

$$-\gamma_i = \frac{-\gamma_{i+1}\rho_i^w(1-\beta)}{(1-\beta) - \gamma_{i+1}\rho_i^c}$$
$$k_i = \frac{\rho_i^w}{\rho_i^c \gamma_i} \left( \rho_i^c \gamma_{i+1} k_{i+1} \right)^{\frac{(1-\beta)}{(1-\beta) - \gamma_{i+1}\rho_i^c}}$$

- And for the Riccati equations define the matrices

$$V_{i+1} = \begin{bmatrix} B^T \Pi_{i+1} B + \frac{1}{\gamma_{i+1}} \Sigma^{-1} & B^T C_1^T \\ C_1 B & -C_1 B \Sigma B^T C_1^T \end{bmatrix},$$
$$K_{i+1} = \begin{bmatrix} B^T \Pi_{i+1} A \\ C_1 A \end{bmatrix}, \quad L_{T-k} = \begin{bmatrix} B^T (\Phi_{i+1} - \Pi_{i+1} \alpha - C_0^T) \\ C_1 \alpha \end{bmatrix}$$

## Appendix: The Riccati equations (2)

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- Define  $C_y$  such that  $y_t = C_y s_t$  and

$$C_{oy} = C_0 + \rho_i^y C_y$$

- Then the Riccati equations are

$$\rho_i^w \Pi_i = A^T \Pi_{i+1} A - K_{i+1}^T V_{i+1}^{-1} K_{i+1}$$

$$\rho_i^w \Phi_i = A^T (\Phi_{i+1} - \Pi_{i+1} \alpha) - C_{oy}^T - K_{i+1}^T V_{i+1}^{-1} L_{i+1}$$

$$\begin{aligned} \rho_i^w \phi_i = & \frac{\delta}{\gamma_{i+1}} + \frac{1}{2\gamma_{i+1}} \log \left| 1 + \gamma_{i+1} \Sigma B^T \Pi_{i+1} B \right| - \frac{1}{2} L_{i+1}^T V_{i+1}^{-1} L_{i+1} \\ & + \left( \phi_{i+1} + \frac{1}{2} \alpha^T \Pi_{i+1} \alpha - \alpha^T \Phi_{i+1} + C_0 B \right) \end{aligned}$$

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