

Global Quantitative Research Monographs

Trend-Following meets Risk Parity

Equities

Global
Quantitative

Extrapolating a price trend

Trend-following strategies are simple trading strategies that take long positions in assets with positive past returns and short positions in assets with negative past returns. They are usually constructed using futures contracts across all asset classes and have historically exhibited great diversification features during dramatic market downturns.

Volatility-Parity

Combining assets from different asset classes into a portfolio requires special attention. The conventional approach is to adjust the size of all positions, so that all assets enter the portfolio with the same volatility. However, this approach ignores pairwise correlations and can turn out to be suboptimal in an environment of increasing correlations.

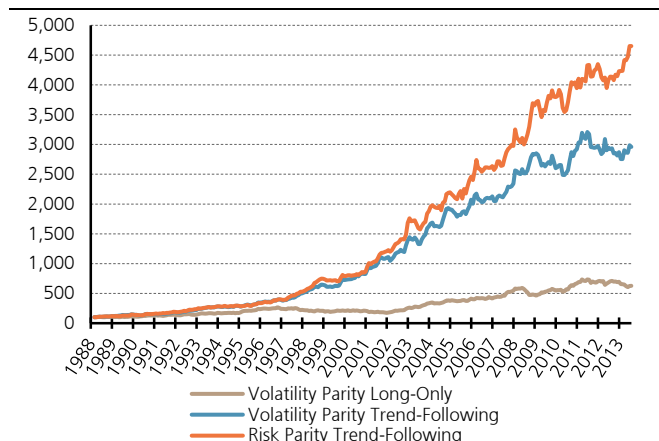
Trend-following has underperformed since 2009

Following a jaw-dropping performance in 2008, a trend-following strategy has failed to generate strong returns over the most recent period. This period has been characterised by large degree of co-movement even across asset classes, with the investable universe being roughly split into the so-called Risk-On and Risk-Off subclasses.

Can a more sophisticated allocation scheme, like Risk-Parity, help?

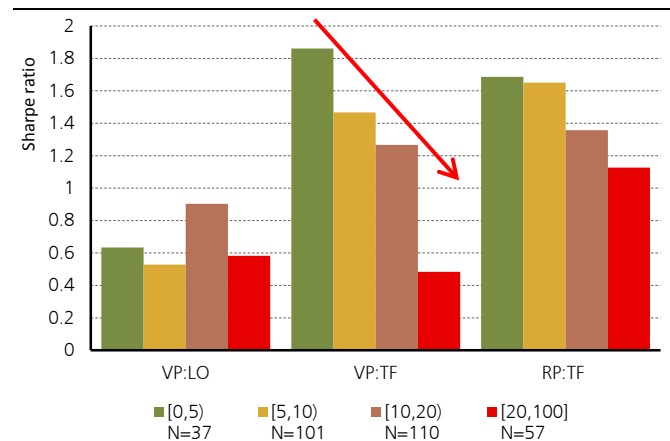
Using our recent extension to the conventionally long-only risk-parity allocation, we construct a long/short trend-following strategy that makes use of risk-parity principles. Not only do we enhance the performance of a simple volatility-parity trend-following strategy, but we show that this enhancement is mostly driven by the more sophisticated weighting scheme in periods of increased correlation. Figures 1 and 2 summarise the findings by presenting cumulative returns and Sharpe ratios across correlation regimes for a volatility-parity long only strategy, a volatility-parity trend-following strategy and a risk-parity trend-following strategy.

Figure 1: Cumulative Returns



Source: UBS Quantitative Research. The figure presents the cumulative returns of a volatility-parity long-only strategy, a volatility-parity 12-month trend-following strategy and a risk-parity 12-month trend-following strategy. All risk measures related to the weighting schemes are estimated using the past 90 days. All strategies are targeting a volatility of 10%. The sample period is from April 1988 to August 2013.

Figure 2: Correlation Event Study



Source: UBS Quantitative Research. The figure presents the annualised Sharpe ratio of a volatility-parity long-only strategy (VP:LO), a volatility-parity trend-following strategy (VP:TF) and a risk-parity trend-following strategy (RP:TF) for four different states of average pairwise correlation: between 0% and 5%, 5% and 10%, 10% and 20% and above 20%. The number of months N for each correlation bucket is shown in the legend. The sample period is from April 1988 to August 2013.

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Introduction & Literature Review

Our 2013 UBS Global Quantitative Conference, which took place in London last April, featured two presentations inspired by two recent academic papers by Baltas and Kosowski (2013a, 2013b) on time-series momentum (also known as trend-following) strategies. Motivated by the client interest, we decided to work on a new research note on trend-following and extend the findings of recent academic literature.

Trend-following is a simple trading strategy that is profitable when assets with positive past returns continue to go up and, similarly, assets with negative past returns continue to fall. In other words, such strategies aim to take advantage of return continuation (serial-correlation) empirical patterns.¹

Extrapolating a price trend

Trend-following strategies are largely employed by systematic funds, like CTAs and managed futures funds² (see Covel 2009 for a broad overview), and are mainly constructed using **futures contracts across all asset classes**³ in an effort to increase diversification. The benefit from using futures contracts is two-fold: first, taking long and short positions using futures contracts is equally straightforward (in contrast, for instance, to using cash equity instruments) and second, the use of futures contracts allows the inclusion of non-equity contracts in the portfolio.

The construction of a trend-following portfolio involves an important challenge that is related to the chosen weighing scheme, given that contracts from different asset classes have very different risk-return profiles (compare a bond future to an equity future or a commodity future). An equal-weight allocation would result in a portfolio that would be dominated in terms of risk by the higher volatility assets, i.e. equities and commodities. Instead, the weighting scheme must make use of the relative "riskiness" of the contracts in order to allocate risk as evenly as possible across all of them.

Challenge: how to allocate risk?

The natural (and slightly naïve for reasons that will be explained later on) solution to the above challenge is to employ volatility-adjusted weights; i.e. assign weights that are inversely proportional to the volatility of each asset so that all assets enter the portfolio with the same ex-ante volatility. For obvious reasons, this scheme is known as the **volatility-parity** scheme. This approach has been followed by the big majority of academic research papers focusing on the topic: e.g. Moskowitz, Ooi and Pedersen (2012), Hurst, Ooi and Pedersen (2012, 2013) and Baltas and Kosowski (2013a, 2013b).

Naïve solution: volatility-adjusted weights

¹ Time-series momentum (trend-following) is structurally different from the conventional cross-sectional winners-minus-losers momentum strategy *a la* Jegadeesh and Titman (1993, 2001). The former is a strategy that takes a position in every asset of the investable universe, is not cash-neutral and, at the extreme, can be in a long-only or short-only state (if all assets have a positive or negative past return respectively); hence, it is a clear bet on the serial correlation of returns. Instead, the latter invests only in the extremes of the cross-section (top vs. bottom decile), it is –in theory– dollar-neutral and its profitability can be either attributed to cross-sectional return dispersion premia or time-series return correlation (the recent paper by Asness, Moskowitz and Pedersen (2013) documents cross-sectional momentum patterns "everywhere"). For an analysis of the relationship between the two momentum strategies see Moskowitz, Ooi and Pedersen (2012) and Clare, Seaton, Smith and Thomas (2014).

² Baltas and Kosowski (2013b) show that futures-based trend-following strategies can explain large part of monthly returns of CTA benchmark indices.

³ Szakmary, Shen and Sharma (2010) study trend-following strategies in commodity markets, Burnside, Eichenbaum and Rebelo (2011) study carry and trend-following strategies in currency markets and finally in two recent papers Clare, Seaton, Smith and Thomas (2013, 2014) study both cross-sectional momentum and trend-following strategies in commodity markets and across broad market indices of different asset classes (equities, bonds, commodities and real estate) from a global asset allocation point of view.

By employing 38 futures contracts from all asset classes (energy, commodities, fixed income, foreign exchange and equities), we construct a volatility-parity trend-following strategy and document its superior performance relative to a long-only equivalent over a long history of more than 25 years (April 1988 to August 2013). The trend-following strategy, by employing long and short positions, can benefit from trending markets (either upwards or downwards) and therefore neutralises the exposure to standard benchmark indices like the MSCI World Index or the DJ UBS Commodity Index. The strategy benefits from the combination of different asset classes and delivers a Sharpe ratio of 1.28 compared to a mere 0.69 for the long-only equivalent. This strong outperformance can render such strategies important diversifying vehicles to include in conventional investment strategies.

So far, so good, but then came 2009. Following an impressive double-digit performance in 2008, the trend-following strategy has consistently delivered very poor performance ever since (see Hurst, Ooi and Pedersen 2012 and Baltas and Kosowski 2013b). This underperformance has received extensive media coverage over the last couple of years. Between January 2009 and August 2013, our volatility-parity trend-following strategy delivers a Sharpe ratio of 0.13 against a respectable Sharpe ratio of 0.54 for the long-only counterparty. What could possibly have gone wrong?

Trend-following has dramatically underperformed since 2009

Following the introduction of the Commodity Futures Modernization Act (CFMA) in 2000, commodities started becoming more correlated to each other as futures markets became accessible to investors as a way to hedge commodity price risk; this increased degree of co-movement in commodity prices is known as the "*financialisation of commodities*".⁴ More generally and more aggressively, following the recent financial crisis in 2008, assets from different asset classes (not just commodities) started becoming more correlated to each other and diversification benefits were dramatically diminished to the extent market participants roughly split the investable universe into two major groups "*Risk On*" and "*Risk Off*".

Increased co-movement

Whatever the underlying reason, the investable universe (the "opportunity set") has exhibited larger co-movement patterns over the last decade. In such an environment, a volatility-parity scheme, by ignoring the covariation between assets, fails to achieve its primary objective, that is, to allocate equal amount of risk to each portfolio constituent. Instead, in an environment of increasing correlations, an asset which correlates less/more with the rest of the universe should be over/under-weighted as it adds/removes diversification. For obvious reasons, we alternatively call volatility-parity as *naïve risk-parity* following Bhansali, Davis, Rennison, Hsu and Li (2012).

The natural extension to a volatility-parity scheme is the so-called *risk-parity* approach; the methodology that calculates portfolio weights so that each constituent contributes the same amount of risk to the overall portfolio, after accounting for any pairwise dynamics. Risk-parity has been rather popular during the last decade due to its remarkable performance (see for example⁵ Anderson,

Equal risk contribution: Risk-Parity

⁴ The financialisation of commodities has recently been a very active research field. Indicatively, see the recent papers by Falkowski (2011), Irwin and Sanders (2011), Tang and Xiong (2012), Basak and Pavlova (2013), Boons, deRoos and Szymanowska (2013), Cheng and Xiong (2013) and Henderson, Pearson and Wang (2013) as well as references therein.

⁵ Both papers by Anderson *et al.* (2012) and Asness *et al.* (2012) employ inverse-volatility weights (i.e. volatility-parity scheme), which they misleadingly call "risk parity" weights, for a stocks and bonds portfolio (2-asset portfolio). To avoid confusion, a risk-parity allocation for two assets degenerates naturally –and mathematically– into a volatility-parity allocation. Hence, their claim for "risk-parity" is valid as a special 2-asset case.

Bianchi and Goldberg 2012 and Asness, Frazzini & Pedersen 2012) and has therefore been a topic of extensive research.⁶

Trend-following investors have been unable to use risk-parity, despite its popularity, because, until very recently, it was an exclusively long-only strategy. It is worth-highlighting that two recent papers by Clare, Seaton, Smith and Thomas (2013, 2014) claim to combine risk-parity with trend-following strategies, but in practice, they only employ conventional volatility-parity schemes that misleadingly call "risk-parity".⁷

Thankfully, our recent Global Quantitative Research Monograph *Understanding Risk Parity* (7 February 2013) extends the long-only paradigm and allows for long and short positions. Motivated by the findings of this research note, we employ the extended risk-parity framework within the construction of trend-following strategies with the hope that the more sophisticated scheme could, if anything, overcome the limitations of a volatility-parity scheme and consequently hedge against drawdowns experienced in high-pairwise-correlation states.

At this stage, it is important to highlight that the profitability of a trend-following strategy depends on two factors: (i) the existence of serial correlation in the return series and (ii) the efficient combination of assets from various asset classes. It is however obvious that the first factor is of utmost importance for the profitability of the strategy; non-existence of price trends cannot be alleviated by a more robust weighting scheme. By changing the volatility-parity scheme into the risk-parity one, we try to address any inefficiency in the risk allocation between portfolio constituents. It cannot be stressed more that in states of the world, where markets are not trending, a different portfolio allocation technique can only do so much.

Having the above in mind, the application of the extended risk-parity methodology on a trend-following strategy proves to be a genuine improvement. Over the entire sample period the risk-parity trend-following strategy increases the Sharpe ratio of the volatility-parity counterparty from 1.28 to 1.46 with the mean return being statistically significantly higher. Using an event study, we show that the suggested methodology significantly reduces the underperformance of the volatility-parity trend-following strategy in months of high average pairwise correlation.

Following this observation and focusing on the recent period of increased pairwise correlation and trend-following underperformance, the risk-parity trend-following scheme achieves in increasing the Sharpe ratio from a mere 0.13 of the volatility-parity counterparty up to 0.58, marginally beating the long-only equivalent (0.54), while however preserving the benefits of trend-following (much lower correlations with traditional benchmark factors compared to the long-only strategy, and about 50% reduction of the long-only maximum drawdown).

To summarise, we do document that the historically impressive outperformance of trend-following strategies has been relatively reduced in the aftermath of the recent financial crisis, but the extent to which this is due to the inefficient weighting scheme can be largely alleviated by introducing more sophisticated risk allocation techniques.

Until very recently, risk-parity was a long-only framework

The risk-parity scheme for a trend-following strategy constitutes genuine improvement

...even after 2009

⁶ The non-exhaustive list of papers includes Bhansali (2011), Lee (2011), Chaves, Hsu, Li and Shakernia (2011, 2012), Bhansali et al. (2012), Lohre, Neugebauer and Zimmer (2012), Jurczenko, Michel and Teiletche (2013), Bernandi, Leippold and Lohre (2013), Fisher, Maymin and Maymin (2013), Lohre, Opfer and Orszag (2013).

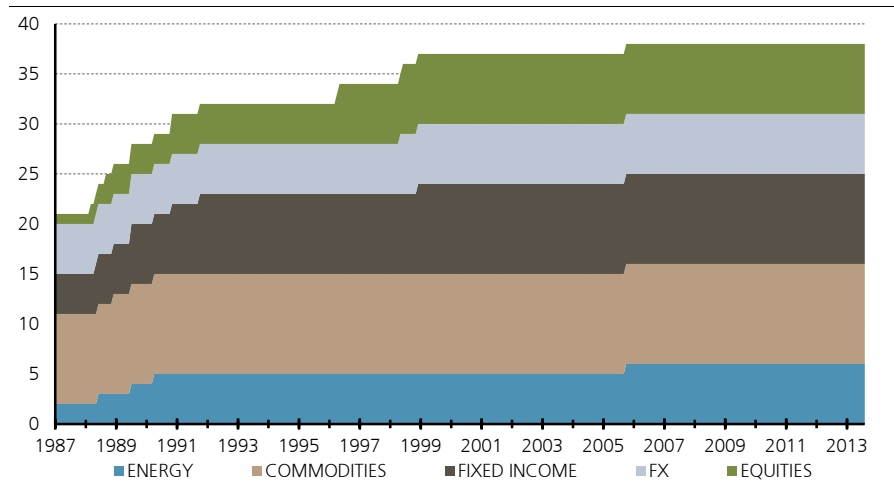
⁷ Adding to the confusion with the risk-parity misnomer, Fisher et al. (2013) call "risk-parity" portfolio what effectively is a volatility-parity portfolio and call "equal risk contribution" portfolio what we call "risk-parity" portfolio.

Data Description

In order to construct trend-following strategies, we use Bloomberg daily closing futures prices for 38 contracts across all asset classes: 6 energy contracts, 10 commodity contracts, 9 fixed income (government bond and short-term interest rate) contracts, 6 foreign exchange contracts and 7 equity index contracts. The choice of the cross-section of contracts follows the constituent list of S&P Systematic Global Macro Index⁸ (consisting of 37 contracts as of September 2013) augmented by FTSE100 futures contract⁹ and is considered to be fairly dispersed both across asset classes and global regions.

Not all contracts have available data from a specific point in time, and for that reason we restrict the sample period to start from January 1987, when all asset classes have at least one contract traded and the cross-section is relatively diverse with 20 contracts being traded in total. Figure 3 presents the evolution in the number of contracts for each asset class over time until August 2013, which is the end of the sample period.

Figure 3: Number of Contracts per Asset Class



Source: UBS Quantitative Research, Bloomberg.

It is important to note that futures contracts have, by their nature, two idiosyncratic features, which do not characterise spot cash equity instruments. First, futures contracts have *finite life* and are only traded for a short period of time before expiration. Second, futures contracts are zero-cost investments and, in theory, no capital is required to initiate a (long or short) position. In practice, entering into a new futures position implies *posting collateral* in form of an *initial margin* payment that is typically a small fraction of the prevailing futures price and a function of the contemporaneous riskiness (as measured by conventional measures such as volatility of value-at-risk) of the underlying entity.

These specific features of futures contracts complicate the back-testing of futures-based trading strategies as *continuous price-series* have to be manufactured and specific assumptions have to be put in place for the calculation of *holding period returns* as illustrated in Baltas and Kosowski (2013a).

38 contracts futures contracts employed across all asset classes

Dealing with the specific features of futures contracts:

- a) finite maturity
- b) margin

⁸ Information and constituent list of the S&P Systematic Global Macro Index is publicly available at the following website: <http://us.spindices.com/indices/commodities/sp-systematic-global-macro-index>

⁹ The results are not driven by inclusion or exclusion of the FTSE100 contract.

Skipping the details at this stage (see Appendix A), we make use of the generic continuous-price series as provided by Bloomberg¹⁰, which are constructed in such a way so that we always trade the most liquid contract, almost always the next-to-mature (also known as the "*front*" contract), and calculate for each futures contract fully-collateralised monthly returns in excess of the prevailing risk-free rate using the formula¹¹:

$$r_{t,t+1} = \frac{F_{t+1} - F_t}{F_t} \quad (1)$$

where F_t and F_{t+1} denote the futures price at the end of month t and $t + 1$ respectively.

Figure 4 presents the entire list of the contracts that we employ, along with summary annualised statistics for the monthly excess returns of each contract for the respective period that each contract is traded. All statistics are calculated using prices expressed in US dollars. What easily stands out is the large cross-sectional dispersion in volatilities, with fixed income contracts exhibiting traditionally the smallest volatilities (Eurodollar 3Mo contract exhibits the smallest annualised volatility of 1%) in contrast to energy contracts that are the most volatile contracts in the cross-section (Natural Gas contract exhibits the largest volatility of 59%). This is also demonstrated in the mean-volatility plane of Figure 5.

Performance-wise, fixed-income contracts and in particular the US bond contracts that have exhibited the best buy-and-hold risk-adjusted performance in the cross-section over the past decades (the well-known "*bond rally*").

The large dispersion in the risk profiles of the contracts is critical when combining them in a portfolio, as a simple equal-weighting scheme would dramatically skew portfolio risk towards the high-volatility assets¹². The methodology section that follows presents the steps we take in order to ex-ante equalise the contribution of risk of each asset to the portfolio.

Large cross-sectional dispersion in volatilities must be carefully addressed

¹⁰ We obtain the Bloomberg generic continuous price-series that are constructed with a ratio-backwards adjustment (screen <GFUT> in Bloomberg provides a number of choices regarding the construction of the generic futures series). The choice of the adjustment is justified in Appendix A.

¹¹ This approach in estimated returns of futures contracts is fairly standard in the academic literature. Indicatively, see de Roon, Nijman and Veld (2000), Moskowitz, Ooi and Pedersen (2012), Baltas and Kosowski (2013b).

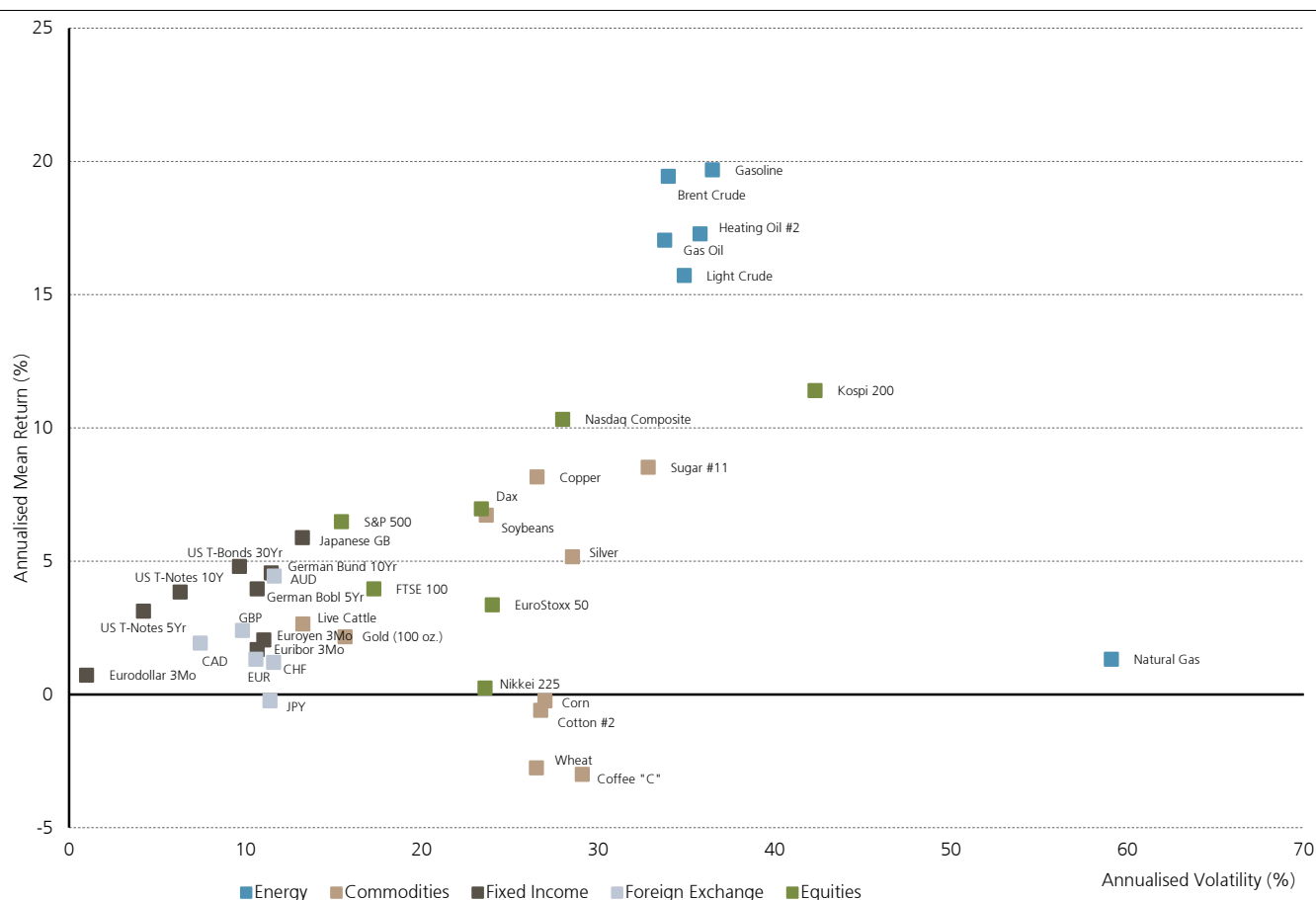
¹² To give a simplistic example, a 50%-50% allocation between an equity futures contract and a bond futures contract would result in a portfolio where 90% of the volatility would be due to the equity contract.

Figure 4: Descriptive Annualised Statistics of Monthly Excess Returns expressed in USD

	From	Obs.	Mean Return (%)	Volatility (%)	Sharpe ratio	Skewness	Kurtosis
ENERGY							
Brent Crude	Jul-88	302	19.44	33.98	0.57	0.55	6.74
Gas Oil	Aug-89	289	17.04	33.77	0.50	0.31	4.46
Gasoline	Nov-05	94	19.68	36.48	0.54	-0.69	5.34
Heating Oil #2	Jan-87	320	17.28	35.78	0.48	1.07	8.47
Light Crude	Jan-87	320	15.72	34.88	0.45	0.28	5.10
Natural Gas	May-90	280	1.32	59.10	0.02	0.87	5.25
COMMODITIES							
Coffee "C"	Jan-87	320	-3.00	29.10	-0.10	0.54	4.13
Copper	Jan-89	296	8.16	26.54	0.31	-0.04	5.44
Corn	Jan-87	320	-0.24	26.99	-0.01	0.75	7.09
Cotton #2	Jan-87	320	-0.60	26.74	-0.02	0.26	3.65
Gold (100 oz.)	Jan-87	320	2.16	15.66	0.14	0.17	4.30
Live Cattle	Jan-87	320	2.64	13.27	0.20	-0.55	6.04
Silver	Jan-87	320	5.16	28.54	0.18	0.21	4.07
Soybeans	Jan-87	320	6.72	23.66	0.28	-0.05	3.84
Sugar #11	Jan-87	320	8.52	32.84	0.26	0.39	3.86
Wheat	Jan-87	320	-2.76	26.50	-0.10	0.39	4.83
FIXED INCOME							
Euro/German Bobl 5Yr	Nov-91	262	3.96	10.67	0.37	0.27	3.90
Euro/German Bund 10Yr	Dec-90	273	4.56	11.47	0.40	0.04	4.16
Japanese GB 10Yr	Jan-87	320	2.04	5.88	0.44	0.65	5.43
US T-Notes 5Yr	Jun-88	303	3.12	4.23	0.74	0.04	3.61
US T-Notes 10Yr	Jan-87	320	3.84	6.30	0.61	0.11	4.19
US T-Bonds 30Yr	Jan-87	320	4.80	9.66	0.50	0.12	4.72
Euribor 3Mo	Jan-99	176	1.68	10.67	0.16	0.09	3.89
Eurodollar 3Mo	Jan-87	320	0.72	1.00	0.72	0.71	6.16
Euroyen 3Mo	Aug-89	289	2.04	11.05	0.18	0.67	6.22
FX							
AUD	Feb-87	319	4.44	11.64	0.38	-0.42	4.73
CAD	Jan-87	320	1.92	7.45	0.26	-0.31	6.69
CHF	Jan-87	320	1.20	11.60	0.10	-0.01	3.58
EUR	Jun-98	183	1.32	10.60	0.12	-0.05	3.82
GBP	Jan-87	320	2.40	9.84	0.24	-0.48	4.79
JPY	Jan-87	320	-0.24	11.40	-0.02	0.51	5.23
EQUITIES							
Dax	Dec-90	273	6.96	23.38	0.01	0.09	3.79
EuroStoxx 50	Jul-98	182	3.36	24.01	0.14	-0.43	3.56
FTSE 100	Mar-88	306	3.96	17.29	0.23	-0.16	4.12
Kospi 200	Jun-96	207	11.40	42.30	0.27	1.01	8.30
Nasdaq Composite	May-96	208	10.32	27.99	0.37	-0.25	3.79
Nikkei 225	Oct-88	299	0.24	23.59	0.01	0.09	3.86
S&P 500	Jan-87	320	6.48	15.45	0.42	-0.78	5.18

Source: UBS Quantitative Research, Bloomberg. All reported statistics are based on USD monthly return series. Mean return, volatility and Sharpe ratio are annualised.

Figure 5: Mean Return-Volatility Plane of Futures Contracts



Source: UBS Quantitative Research. The figure presents the mean-volatility plane (using unconditional annualised measures) for each futures contract in the dataset. Contracts of the same asset class are presented with the same colour. All statistics are calculated in USD. The sample period is variable across contracts and starts at the month indicated in Figure 4. The end of the sample period is August 2013.

Methodology

Trend-Following 101

Let N_t denote the number of available futures contracts at time t . A trend-following (TF , henceforth) strategy involves taking a long or short position on each asset i , based on the sign of the *local-currency* past return over a prescribed lookback period. If x_t^i denotes the *signed* amount of USD invested in asset i at time t (i.e. $x_t^i > 0$ for long positions and similarly $x_t^i < 0$ for short positions), the return of the strategy is calculated as the weighted average of the individual holding period returns of the contracts:

$$r_{t,t+1}^{TF} = \sum_{i=1}^{N_t} \frac{x_t^i}{\sum_{j=1}^{N_t} |x_t^j|} \cdot r_{t,t+1}^i = \sum_{i=1}^{N_t} w_t^{Net,i} \cdot r_{t,t+1}^i \quad (2)$$

where $w_t^{Net,i} = \frac{x_t^i}{\sum_{j=1}^{N_t} |x_t^j|}$ denotes the **net weight** invested in asset i at time t . The net weights do not in practice add up to 100%, since they can take either positive or negative values. The type of the position is determined by the sign of the past return over a lookback period of J months. Thus, the net weight can be written as the product of the **gross (or absolute) weight** and the sign of past return:

$$w_t^{Net,i} = \text{sign}(r_{t-J,t}^i) \cdot w_t^{Gross,i} \quad (3)$$

Trivially, $\sum_{i=1}^{N_t} w_t^{Gross,i} = 100\%$. Substituting equation (3) back into (2) yields:

$$r_{t,t+1}^{TF} = \sum_{i=1}^{N_t} \text{sign}(r_{t-J,t}^i) \cdot w_t^{Gross,i} \cdot r_{t,t+1}^i \quad (4)$$

The above formulation treats long and short positions equally as it is assumed that the opening of any position requires posting a collateral equal to the prevailing futures price of the contract (*fully-collateralised positions*).¹³

As a final step it must be noted that trend-following strategies are normally implemented on a constant-volatility¹⁴ (CV) basis by targeting a certain level of volatility σ_{TGT} . Thus, the generalised formulation of a $CV:TF$ strategy is given by:

$$r_{t,t+1}^{CV:TF} = \frac{\sigma_{TGT}}{\sigma_t^{TF}} \cdot \sum_{i=1}^{N_t} w_t^{Net,i} \cdot r_{t,t+1}^i \quad (5)$$

where σ_t^{TF} denotes an estimate of running realised volatility of the *unlevered* trend-following strategy as formulated in equations (2) and (4). This formulation introduces **time-varying leverage** equal to the ratio $\sigma_{TGT}/\sigma_t^{TF}$.

Constructing the generic trend-following strategy:

→ Trading signal is calculated using local currency returns

Running the strategy with a target (constant) level of volatility

→ Employ leverage

¹³ This is in line with the mechanics of futures markets, where entering a futures position, either long or short, requires *posting* a collateral equal to the initial margin; hence, there is initial cost involved even though futures are in theory unfunded investment instruments.

¹⁴ For further information of constant-volatility strategies, see our Global Quantitative Research Monographs *Understanding Volatility Targeting* (4 October 2011) and *Extending Volatility Targeting* (3 September 2013). Similar technique has been employed by Barroso and Santa-Clara (2013) and Daniel and Moskowitz (2013), who focus on cross-sectional winners-minus-losers momentum strategies.

Volatility Parity (Naïve Risk Parity)

Given that different futures contracts have very different risk profiles (as presented in Figure 4), it is critical to determine a weighting scheme that assigns a weight to every asset that is a function of its underlying *riskiness*, in an effort to construct a strategy with a fairly balanced distribution of risk across assets and asset classes.

Following Moskowitz *et. al* (2012) and Baltas and Kosowski (2013a, 2013b), one obvious choice for the weight of each asset is to be inversely proportional to a measure of historical realised volatility of the asset. This choice implies that all assets enter the portfolio with the same level of ex-ante volatility. For that reason, this strategy can be called a **Volatility-Parity Trend-Following (VP:TF)** strategy. The signed amount of USD units invested in asset i at time t is given by:

$$x_t^{VP:TF,i} = \frac{\text{sign}(r_{t-J,t}^i)}{\sigma_t^i} \quad (6)$$

It is important to highlight that the trend-following signal, the sign of the past returns, is generated using local-currency prices, whereas the volatility of the asset is calculated using USD prices. This is done because, on one hand, we want to identify local-currency trending behaviour, but on the other hand the strategy would need to quantify risk in terms of USD, hence the amount of USD invested in each asset has to be deduced from a USD-expressed measure of risk.

Substituting back into equation (5) yields the return series of the strategy:

$$r_{t,t+1}^{VP:TF} = \underbrace{\frac{\sigma_{TGT}}{\sigma_t^{TF}}}_{\text{Constant Volatility}} \sum_{i=1}^{N_t} \underbrace{\text{sign}(r_{t-J,t}^i)}_{\text{Long / Short Positions}} \cdot \underbrace{\frac{(\sigma_t^i)^{-1}}{\sum_{i=1}^{N_t} (\sigma_t^i)^{-1}}}_{\text{Inverse Volatility Weighted}} \cdot r_{t,t+1}^i \quad (7)$$

It is critical to observe that the suggested portfolio construction methodology *does not account* for pairwise correlations between the available assets (except indirectly in calculating σ_t^{TF}) or, in other words, makes the assumption that all pairwise correlations are equal to zero and therefore the variance-covariance matrix is diagonal. If this were the case, then the portfolio would be a true risk-parity portfolio (to be defined at a later stage); however, this assumption is not always true in practice. For this reason, we also call the **VP:TF** strategy the *Naïve Risk-Parity Trend-Following* strategy.

Equation (7) gives rise to a natural **Long-Only (LO)** benchmark, if all positions are constantly long. The resulting **VP:LO** strategy is effectively the volatility-weighted portfolio of the assets targeted at the desired level of overall portfolio volatility:

$$r_{t,t+1}^{VP:LO} = \frac{\sigma_{TGT}}{\sigma_t^{LO}} \sum_{i=1}^{N_t} \underbrace{(+1)}_{\text{Long Only}} \cdot \frac{(\sigma_t^i)^{-1}}{\sum_{i=1}^{N_t} (\sigma_t^i)^{-1}} \cdot r_{t,t+1}^i \quad (8)$$

In what follows, all trend-following signals are generated at the end of each month, but strategies are rebalanced daily in order to achieve the required target level of volatility of 10%. The turnover implications of this are discussed in the last section of this research paper.

Volatility-adjusted weights

Volatility-Parity is optimal when Variance-Covariance matrix is diagonal

Long-Only Benchmark Strategy

Performance Evaluation

The Effect of the Lookback Period

We start our empirical analysis by looking into the performance of the trend-following strategy as a function of the lookback period that is used to generate momentum signals. Figures 6 to 11 present the ex-post Sharpe ratio of the long-only and the volatility-parity trend-following strategies across each asset class and across the entire universe of available contracts, for lookback periods that range between 1 and 24 months¹⁵.

It is visually straightforward to identify that a trend-following strategy with a lookback period of 12 months is a top 3 performer across any asset class and the best performing strategy across all contracts. This result comes as no surprise; the very large majority of academic literature on cross-sectional momentum (Jegadeesh and Titman 1993, 2001, Asness *et al.* 2013) and time-series momentum (Moskowitz *et al.* 2012, Baltas and Kosowski 2013b) agree that a 12-month lookback period results in the best ex-post performance.

From a more philosophical point of view, a 12-month cycle can be justified either from the nature of the contracts and the fundamental supply and demand forces of the market, e.g. agriculture prices are expected to exhibit seasonality (which could potentially be the reason for the existence of a 2-year momentum cycle in commodities in Figure 8) or because of human behaviour¹⁶; if every investor were a trend-follower and goods were available with infinite supply, prices would move monotonically, hence giving rise to the identified serial correlation/return continuation patterns.

Whichever might be the case, and even if our choice may suffer from a look-ahead bias, we argue that a 12-month lookback period has always been and will naturally remain one of the typical lookback periods that investors pay attention to. With this in mind, we decide to fix the lookback period to 12 months for the remainder of the research paper.

Finally, contrasting the performance of trend-following strategies with their long-only counterparts, it is worth-highlighting that 12-month trend-following almost always delivers substantially larger Sharpe ratios with the only exception being the fixed income universe. To be precise, in the fixed income universe, it has not really been the trend-following strategy that has underperformed (delivering a Sharpe ratio that is comparable to that of the other asset classes), but instead, the long-only strategy has performed substantially better than any other long-only strategy that we have constructed. The reason is rather obvious and relates to the very good overall performance of this particular asset class over the past decades; the so-called "*bond-rally*". Further evidence of these effects follows in the next sections.

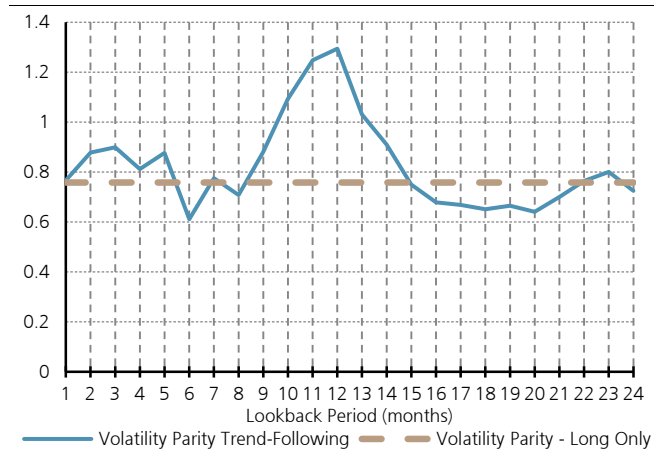
How to justify a 12-month lookback period?

The "bond-rally"

¹⁵ The sample period starts in April 1989, because this is the first month that a 24-month trading signal can be generated.

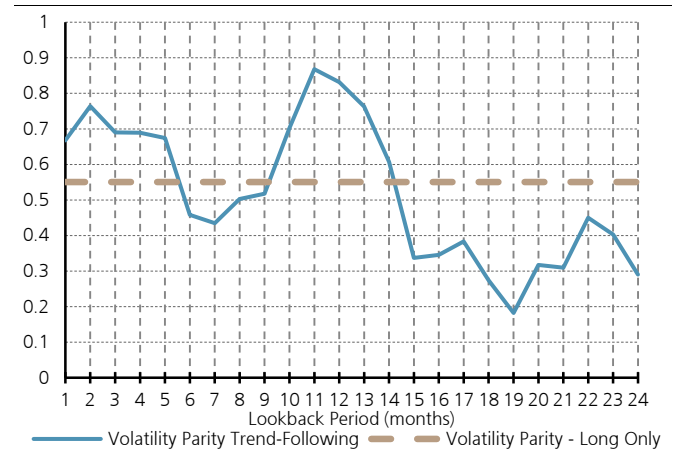
¹⁶ For example see Barberis, Shleifer and Vishny (1998), Daniel, Hirshleifer and Subrahmanyam (1998), Hong and Stein (1999), Frazzini (2006). Alternatively Ilmanen (2011) contains a broad overview.

Figure 6: Sharpe Ratios – All Contracts



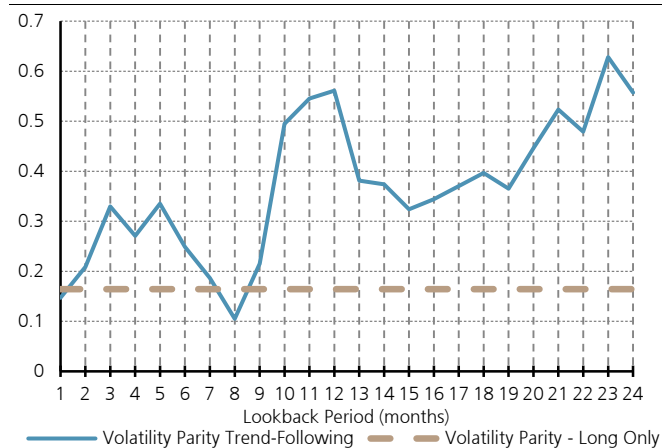
Source: UBS Quantitative Research. Relationship between the Sharpe ratio and the lookback period for monthly volatility-parity strategies using all available futures contracts. For comparison, the figure includes the Sharpe ratio of a long-only constant volatility strategy. Sample Period: April-1989 to August-2013.

Figure 7: Sharpe Ratios – Energy Contracts



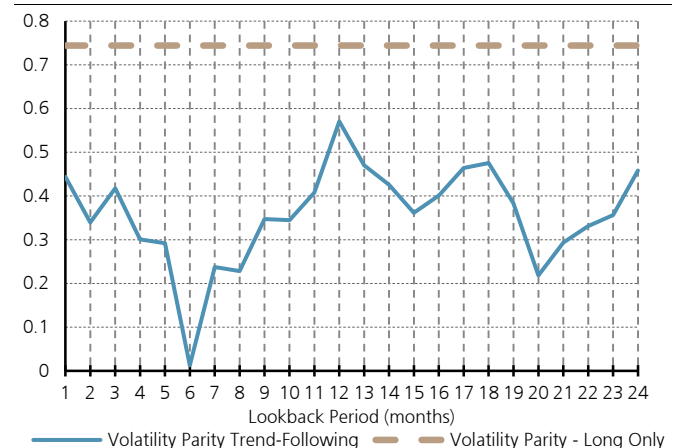
Source: UBS Quantitative Research. Relationship between the Sharpe ratio and the lookback period for monthly volatility-parity strategies using energy futures contracts. For comparison, the figure includes the Sharpe ratio of a long-only constant volatility strategy. Sample Period: April-1989 to August-2013.

Figure 8: Sharpe Ratios – Commodity Contracts



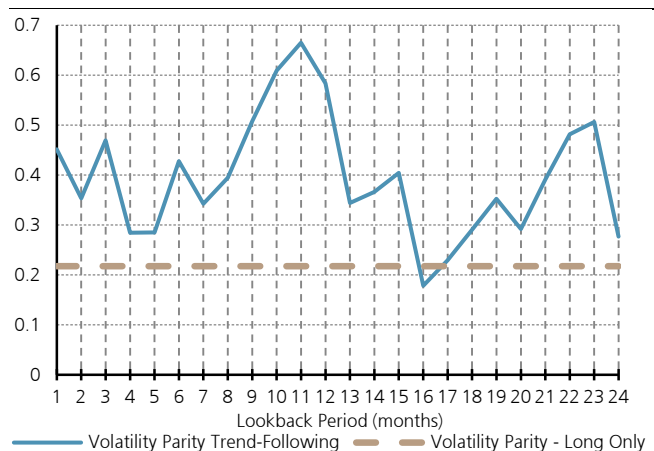
Source: UBS Quantitative Research. Relationship between the Sharpe ratio and the lookback period for monthly volatility-parity strategies using commodity futures contracts. For comparison, the figure includes the Sharpe ratio of a long-only constant volatility strategy. Sample Period: April-1989 to August-2013.

Figure 9: Sharpe Ratios – Fixed Income Contracts



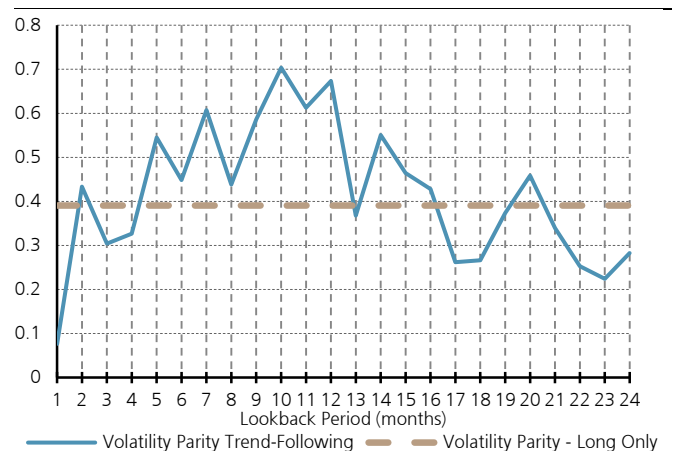
Source: UBS Quantitative Research. Relationship between the Sharpe ratio and the lookback period for monthly volatility-parity strategies using fixed income futures contracts. For comparison, the figure includes the Sharpe ratio of a long-only constant volatility strategy. Sample Period: April-1989 to August-2013.

Figure 10: Sharpe Ratios – FX Contracts



Source: UBS Quantitative Research. Relationship between the Sharpe ratio and the lookback period for monthly volatility-parity strategies using foreign exchange futures contracts. For comparison, the figure includes the Sharpe ratio of a long-only constant volatility strategy. Sample Period: April-1989 to August-2013.

Figure 11: Sharpe Ratios – Equity Contracts



Source: UBS Quantitative Research. Relationship between the Sharpe ratio and the lookback period for monthly volatility-parity strategies using equity futures contracts. For comparison, the figure includes the Sharpe ratio of a long-only constant volatility strategy. Sample Period: April-1989 to August-2013.

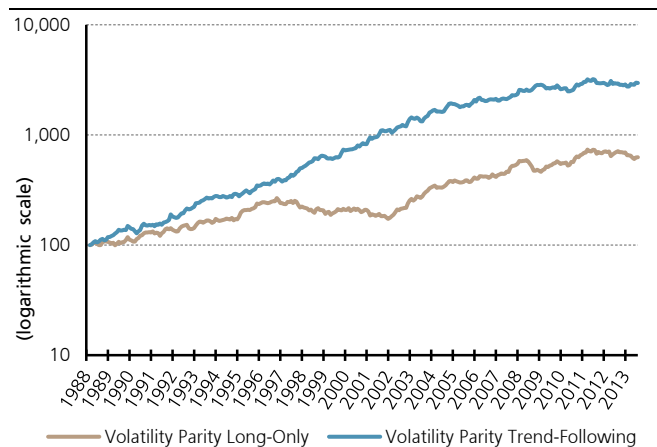
The Benefits of Trend-Following

The cumulative returns of a volatility-parity long-only strategy and a trend-following strategy with 10% target volatility are presented in Figure 12. Additionally, Figure 13 presents the amount of gross leverage employed by each strategy in an effort to achieve an ex-ante constant volatility profile; this leverage factor is effectively equal to the ratio $10\%/\sigma_t$, where σ_t denotes the running volatility (measured over the past 60 business days) of the long-only and the trend-following unlevered strategies.

The outperformance of the trend-following strategy is largely pronounced. The gross leverage employed reaches values as high as 7x, when the long-only equivalent rarely exceeds 4x. The amount of leverage is systematically larger for the trend-following strategy, because the combination of long and short positions reduces the overall volatility of the unadjusted strategy, which, in turn, implies larger amount of leverage in relative terms, in order to achieve the required level of target volatility.

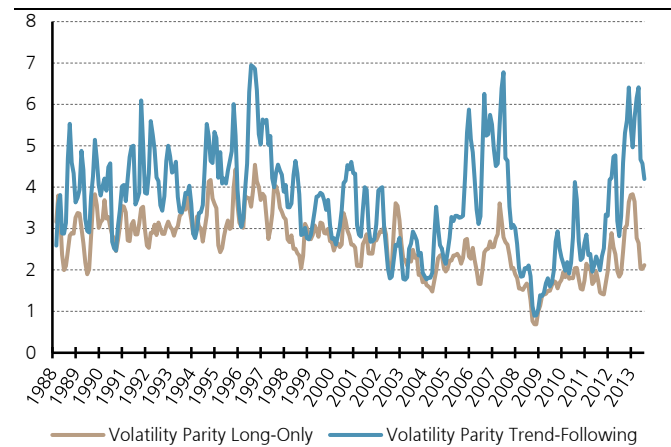
Trend-following largely outperforms the long-only benchmark

Figure 12: Cumulative Returns



Source: UBS Quantitative Research. The figure presents the cumulative returns of a long-only strategy and a trend-following strategy (with 12-month lookback period), both employing a volatility-parity weighting scheme estimated using the past 90 days. The scale is logarithmic. The sample period is from April 1988 to August 2013.

Figure 13: Leverage



Source: UBS Quantitative Research. The figure presents the leverage factor of the volatility-parity long-only strategy and the volatility-parity 12-month trend-following strategy. This is effectively the ratio of the target level of volatility, 10%, to the 60-day running realised volatility of each unlevered strategy. The sample period is from April 1988 to August 2013.

Performance statistics as well as correlations with major benchmark indices across asset classes are presented in Figure 14. We summarise the main findings below:

- Trend-following almost doubles the Sharpe ratio and the Sortino ratio of its long-only counterpart (1.28 vs. 0.69 and 2.70 vs. 1.70).
- The trend-following strategy is successful across all asset classes and the achieved Sharpe ratios range between 0.58 and 0.71.
- However, the largest benefit is related to the combination of different asset classes in a single portfolio, the trend-following portfolio.
- The long-only strategy bears, by construction, strong directional bets, as deduced by the large correlation across any of the chosen benchmark indices (Panel B). From a risk analysis point of view, all these correlations are translated into beta exposures to the underlying factors.
- On the contrary, the trend-following strategies exhibit very small, if not insignificant, exposure to all benchmark indices. This justifies the use of such strategies as a means of diversification.

- The only large correlation between the trend-following strategies and the benchmark indices is exhibited between the fixed-income strategy and the JP Morgan Global Bond Index. The logic is very simple: given the bond rally, the fixed-income strategy employs many long positions over time, hence exhibits large correlation with the (long-only) bond index itself.
- Finally, going back to the cumulative returns of Figure 12, it can be argued that the performance of the trend-following strategy has diminished over time especially during the most recent post-crisis period. We will return to this period of underperformance in later sections in order to investigate its source.

Figure 14: Volatility-Parity Long-Only versus Trend-Following Strategies

Panel A: Performance Statistics

	<i>VP: LO</i>	<i>VP: TF</i>					
	ALL	ALL	NRG	CMDTY	FI	FX	EQ
Ann. Geometric Mean (%)	7.49	14.26	8.03	6.01	5.98	6.26	6.55
T-statistic (Newey-West)	3.19	6.54	3.29	3.83	2.55	3.14	3.18
Ann. Volatility (%)	11.52	10.96	11.84	10.47	11.06	11.07	10.72
Skewness	-0.14	0.38	0.77	0.23	0.33	0.45	0.23
Kurtosis	3.04	3.30	5.61	3.68	3.71	5.12	3.13
Max Drawdown (%)	35.22	14.20	25.08	26.94	24.89	26.57	19.76
Sharpe Ratio (annualised)	0.69	1.28	0.71	0.61	0.58	0.60	0.65
Sortino Ratio (annualised)	1.11	2.70	1.32	1.04	0.99	1.03	1.10
Calmar Ratio	0.21	1.00	0.32	0.22	0.24	0.24	0.33

Panel B: Correlations with Benchmark Factors

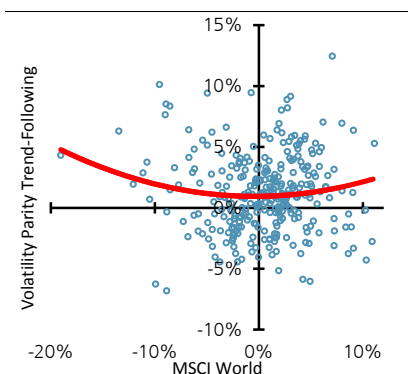
	<i>VP: LO</i>	<i>VP: TF</i>					
	ALL	ALL	NRG	CMDTY	FI	FX	EQ
<i>Commodity Benchmarks:</i>							
- DJ UBS Commodity	0.69	0.10	0.13	0.09	0.05	0.02	-0.06
- S&P GSCI Commodity	0.58	0.11	0.24	0.00	0.03	0.00	0.01
<i>Fixed Income Benchmarks:</i>							
- JPM Global Bond Index	0.69	0.31	-0.02	0.06	0.59	0.10	-0.04
<i>FX Benchmarks:</i>							
- Trade-Weighted USD	-0.54	-0.14	-0.04	0.01	-0.21	-0.08	0.04
- USD/CHF	-0.59	-0.21	-0.02	-0.04	-0.39	-0.11	0.08
- USD/JPY	-0.45	-0.18	0.05	-0.02	-0.42	-0.06	0.05
<i>Equity Benchmarks:</i>							
- MSCI World (DM)	0.52	-0.01	0.01	-0.05	-0.05	-0.07	0.03
- MSCI Emerging Markets	0.37	-0.05	0.05	0.02	-0.15	-0.16	0.03

Source: UBS Quantitative Research. The figure reports in Panel A various performance statistics using monthly returns for the volatility-parity long-only strategy (*VP: LO*) across all contracts and for the volatility-parity trend-following (with 12-month lookback period) strategy (*VP: TF*) across all contracts and across asset classes; energy (NRG), commodity (CMDTY), fixed-income (FI), foreign exchange (FX) and equity (EQ). The t-statistic of the mean return is calculated using Newey and West (1987) standard errors. The Sortino ratio is defined as the annualised arithmetic mean return over the annualised downside volatility. The Calmar ratio is defined as the annualised geometric mean return over the maximum drawdown. Panel B presents correlations between the aforementioned strategies and various benchmarks across all asset classes. All correlation estimates with absolute value larger than 0.50 are presented in red. The sample period for all statistics is from April 1988 to August 2013, except for correlations with DJ UBS Commodity index, which is available since February 1991.

Given that MSCI World tracks the performance of the global equity market, it is worth highlighting the almost-zero correlation that it exhibits with the volatility-parity trend-following strategy (point estimate is -0.01). This seemingly uncorrelated pair bears interesting non-linear (higher-order) correlation dynamics as shown in the scatterplot of Figure 15. By employing long and short positions, the trend-following strategy can benefit from both extreme upward and downward market movements, hence exhibiting important diversification features.

Finally, some further empirical evidence on the correlation structure between long-only and trend-following strategies is presented in Figure 16. The upper-left (lower-right) part of the matrix presents the correlations within the long-only (trend-following) spectrum of strategies. Overall, trend-following strategies appear to be less correlated to each other in comparison to their long-only counterparts. This makes the diversification benefit from the combination of all asset classes greater in the trend-following world.

Figure 15: MSCI World vs. $VP:TF$



Source: UBS Quantitative Research. Scatterplot between MSCI World returns and $VP:TF$ returns along with a quadratic least-squares fit. The sample period is from April 1988 to August 2013.

Figure 16: Correlation Matrix between Long-Only and Trend-Following Strategies

		Long-Only						Trend-Following					
		ALL	NRG	CMDTY	FI	FX	EQ	ALL	NRG	CMDTY	FI	FX	EQ
Long-Only	ALL	1.00											
	NRG	0.52	1.00										
	CMDTY	0.57	0.15	1.00									
	FI	0.59	0.02	0.09	1.00								
	FX	0.81	0.31	0.36	0.53	1.00							
	EQ	0.54	0.16	0.24	0.12	0.34	1.00						
Trend-Following	ALL	0.26	0.13	0.01	0.33	0.19	0.05	1.00					
	NRG	0.07	0.25	-0.03	-0.06	0.00	0.01	0.44	1.00				
	CMDTY	0.04	-0.04	0.03	0.08	0.03	-0.01	0.54	0.05	1.00			
	FI	0.36	0.02	0.10	0.67	0.29	-0.06	0.49	-0.01	0.02	1.00		
	FX	0.10	0.09	0.01	0.10	0.13	-0.06	0.44	0.00	0.14	0.04	1.00	
	EQ	0.03	0.01	-0.08	-0.05	-0.01	0.18	0.54	0.13	0.16	0.02	0.21	1.00

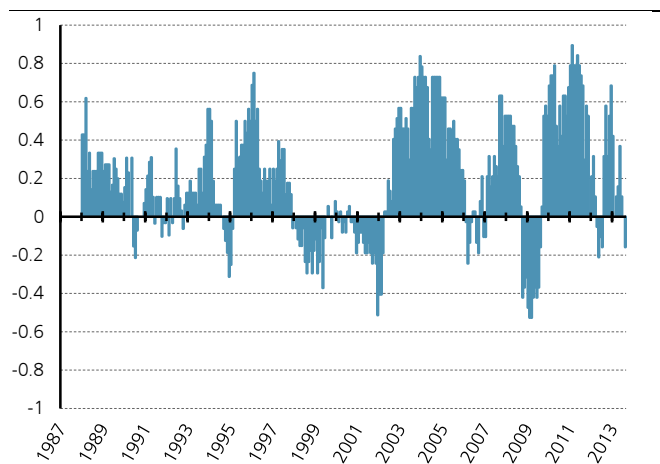
Source: UBS Quantitative Research. The figure presents the correlation matrix between long-only and trend-following (with 12-month lookback period) strategies that make use of a volatility-parity weighting scheme estimated using the past 90 days. The correlation estimates between long-only and trend-following strategies of the same asset class are presented in bold [diagonal of the lower left part of the matrix]. All correlation estimates with absolute value larger than 0.50 are presented in red. The sample period is from April 1988 to August 2013.

More interestingly, the bottom-left shaded area of the matrix presents the correlations of all possible pairs between long-only and trend-following strategies and documents the very low cross-correlations between the two spectrums of strategies. The only pair that exhibits a relatively large correlation is, expectedly, the two strategies consisting exclusively of fixed income contracts.

In order to investigate further the benefits of employing both long and short positions in a futures portfolio, Figures 18 to 23 present the overall net position across the entire universe and within asset classes as instructed by the trend-following rule at the end of each month. The net position calculation is simply the sum of the available contracts including the sign of the position (long or short) normalised by the total number of contracts available¹⁷. Moreover, Figure 23 reports the proportion of time over the entire sample period that each trend-following strategy is net long.

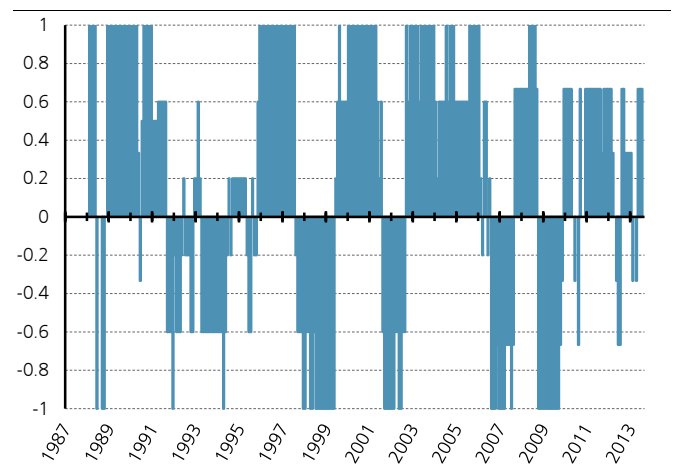
¹⁷ The calculation does not involve portfolio weights, but instead only calculates the net number of available assets that comprise the portfolio.

Figure 17: Net Position – All Contracts



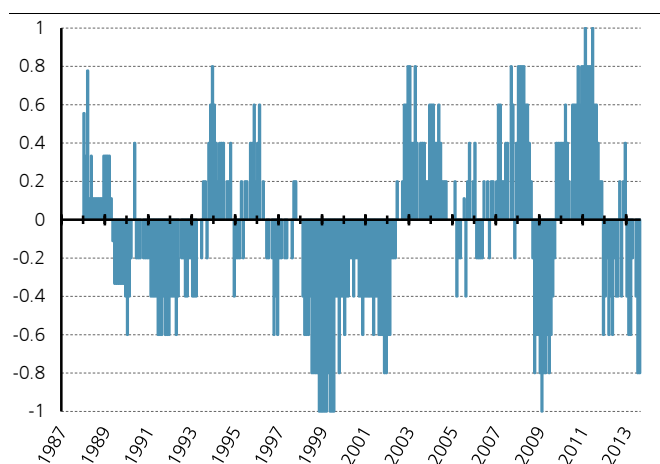
Source: UBS Quantitative Research. Net position of a trend-following rule with a 12-month lookback period using all available futures contracts. The net position is expressed in terms of the number of available contracts at the end of each month.

Figure 18: Net Position – Energy Contracts



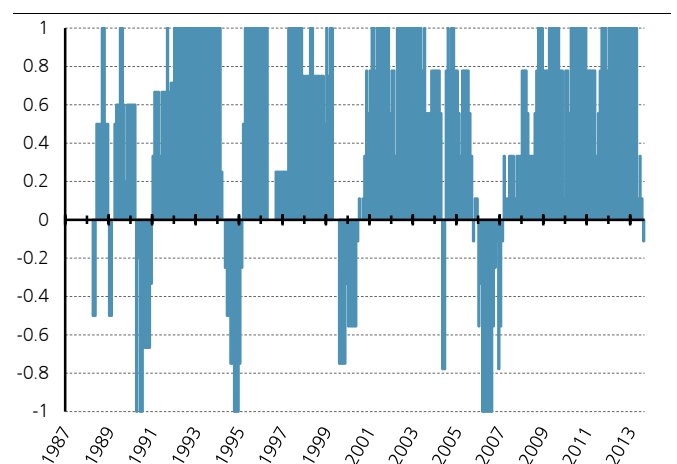
Source: UBS Quantitative Research. Net position of a trend-following rule with a 12-month lookback period using energy futures contracts. The net position is expressed in terms of the number of available contracts at the end of each month.

Figure 19: Net Position – Commodity Contracts



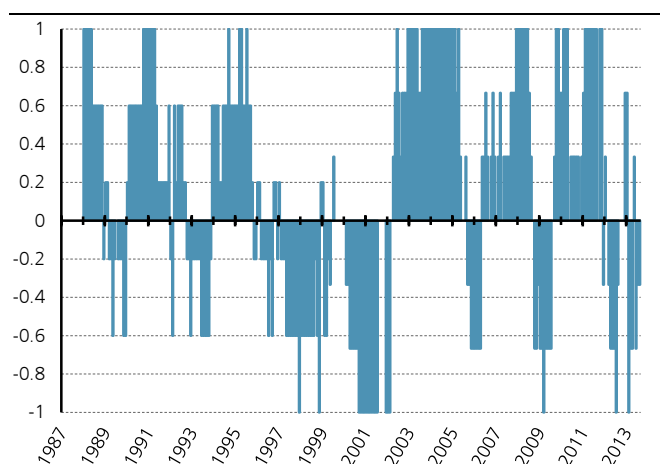
Source: UBS Quantitative Research. Net position of a trend-following rule with a 12-month lookback period using commodity futures contracts. The net position is expressed in terms of the number of available contracts at the end of each month.

Figure 20: Net Position – Fixed Income Contracts



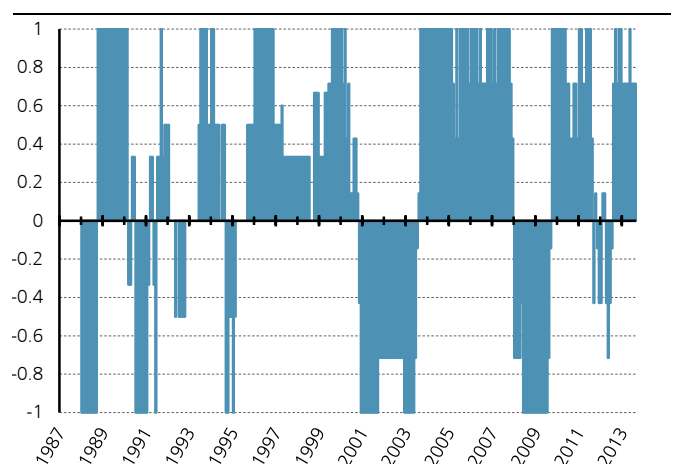
Source: UBS Quantitative Research. Net position of a trend-following rule with a 12-month lookback period using fixed income futures contracts. The net position is expressed in terms of the number of available contracts at the end of each month.

Figure 21: Net Position – FX Contracts



Source: UBS Quantitative Research. Net position of a trend-following rule with a 12-month lookback period using foreign exchange futures contracts. The net position is expressed in terms of the number of available contracts at the end of each month.

Figure 22: Net Position – Equity Contracts



Source: UBS Quantitative Research. Net position of a trend-following rule with a 12-month lookback period using equity index futures contracts. The net position is expressed in terms of the number of available contracts at the end of each month.

The points that are worth-highlighting are listed below:

- All asset classes except commodities appear to switch between long-only and short-only states. This fact denotes that pairwise correlations tend to be relatively large at the intra-asset-class level.
- Contrary to the above, the strategy across all contracts is never entirely long-only or short-only. This is because the different asset classes tend to exhibit lower inter-asset-class correlations and therefore their combination benefits from the implied diversification benefit.
- The fixed income strategy is net long most of the time (79%). This comes as no surprise. The bond rally (decreasing interest rates) has caused the trend-following strategy to load positively on fixed income instruments for most of the sample period. This explains the high levels of correlation related to the fixed income universe in Figures 14 and 16.

Figure 23: Net Long Statistics

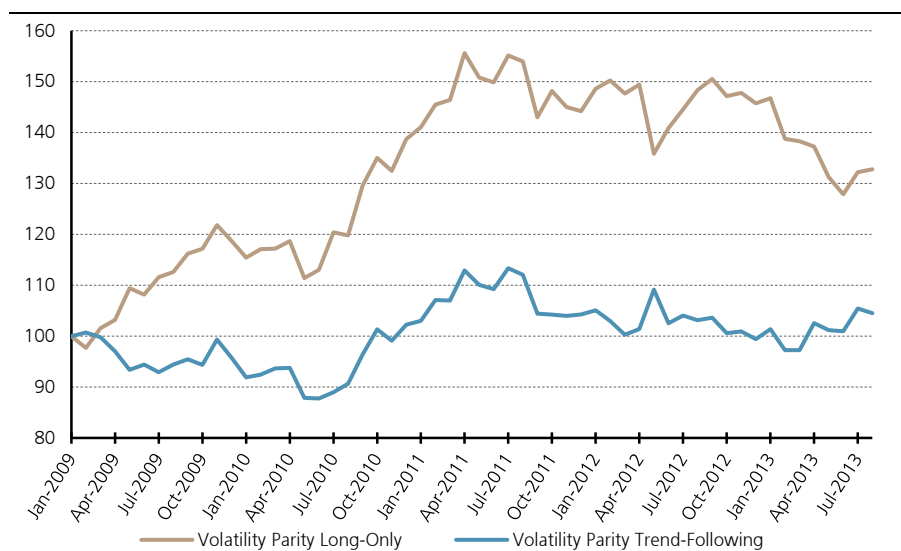
Asset Class	Proportion of Time
All Contracts	69%
Energy	61%
Commodities	40%
Fixed Income	79%
FX	54%
Equities	63%

Source: UBS Quantitative Research. The figure presents the proportion of time (percentage of months) that each strategy is net long. Sample period: April 1988 – August 2013.

...and then Trend-Following stopped working

Even though trend-following has been very successful over the entire sample period tested above (April 1988 to August 2013), delivering superior risk-adjusted performance in comparison to its long-only counterparty, it has dramatically underperformed after the recent financial crisis of 2008. Figure 24 presents the cumulative returns of the long-only strategy and the volatility-parity trend-following strategy with 10% target volatility for the period between January 2009 and August 2013.

Figure 24: Trend-Following Recent Underperformance (2009 onwards)



Source: UBS Quantitative Research. The figure presents the cumulative returns of a long-only strategy and a trend-following strategy (with 12-month lookback period), both employing a volatility-parity weighting scheme estimated using the past 90 days. The sample period is from January 2009 to August 2013.

Contrary to the full sample statistics, trend-following has remained flat over the most recent period delivering a mere Sharpe ratio of 0.13 compared to a ratio of 0.54 for the long-only strategy.

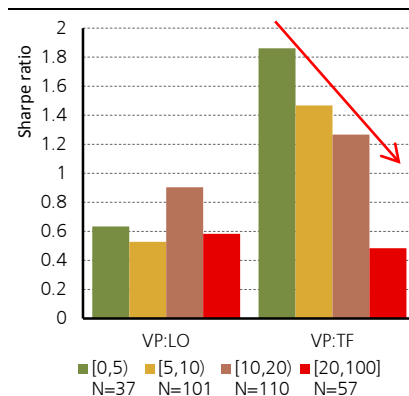
What could possibly have gone wrong?

There is evidence (to be presented in the next section) that the degree of co-movement has increased over the last decade and in particular following the recent credit crisis. In an environment of increased correlation, diversification benefits diminish and assets are effectively clustered into "more-risky" and "less-risky" buckets, commonly referred to as a "*Risk-On/Risk-Off*" environment. In such an environment, the covariance matrix of the assets is far from diagonal and the argument that volatility-parity is not an optimal risk allocation methodology becomes more compelling.

In order to empirically investigate this hypothesis, we construct a correlation event study that is presented in Figure 25. In particular, we group all months of the dataset in four average-pairwise-correlation buckets (from 0% to 5%, from 5% to 10%, from 10% to 20% and from 20% and above) and evaluate the performance of the long-only and the trend-following strategies in terms of Sharpe ratio. The evidence is clear. The performance of the volatility-parity trend-following strategy drops dramatically when the level of average pairwise correlation deviates significantly away from zero and into the positive territory.

Could this be due to the inefficient volatility-parity weighting scheme that has been employed or are trend-following patterns less pronounced in higher-correlation states? In the next section, we introduce *risk-parity* and try to identify whether the inefficient volatility-parity scheme could be one of the reasons for the recent underperformance of the trend-following strategy.

Figure 25: Correlation Event Study



Source: UBS Quantitative Research. The figure presents the annualised Sharpe ratio of a volatility-parity long-only strategy (*VP:LO*) and a trend-following strategy (with a 12-month lookback period) (*VP:TF*) for four different states of average pairwise correlation: between 0% and 5%, 5% and 10%, 10% and 20% and above 20%. The number of months *N* for each correlation bucket is shown in the legend. The sample period is from April 1988 to August 2013.

Rendering Trend-Following Robust

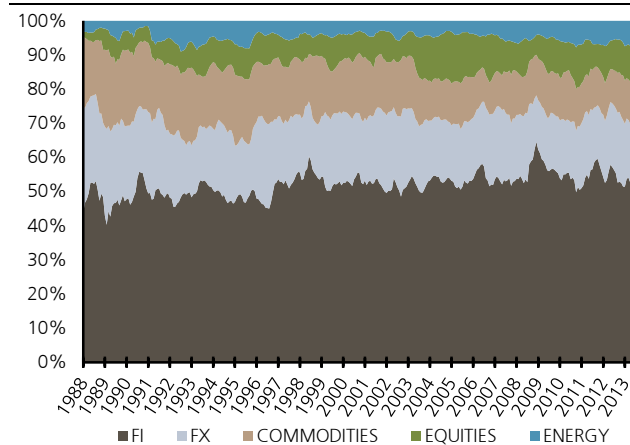
Volatility Parity ignores Correlation Structure

Volatility-parity can only successfully distribute the overall risk evenly to all portfolio constituents when all pairwise correlations are equal to zero, or in other words, when the variance-covariance matrix is diagonal. In practice, this is a very strict assumption that is never satisfied.

To further elaborate, Figure 26 presents the time series of the sum of gross weights of each asset class, i.e. $\sum_j |w_t^j|$ for each asset class group and Figure 27 presents the time series of the weighted contribution to risk of each asset class, normalised by the total risk (the so-called *percentage contribution to risk*), i.e. $\sum_j w_t^j \cdot MCR_t^j / \sigma_t$ for each asset class group; *MCR* denotes the so-called *marginal contribution to risk* and is defined as the partial derivative of portfolio volatility with respect to the weight of each asset (more details follow in the next section).

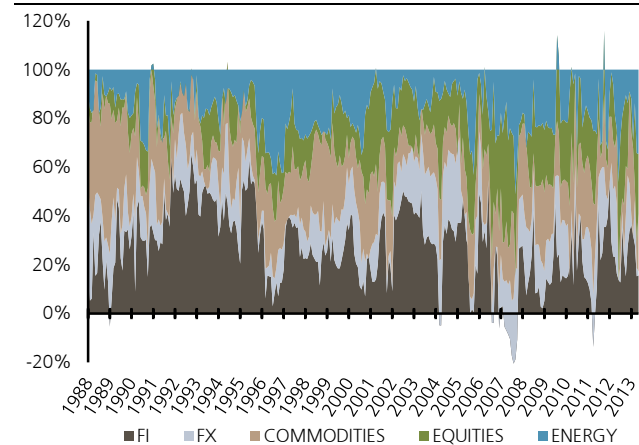
Given the large persistence of volatility as a risk statistic, the volatility-parity weighting scheme results in fairly stable weight allocation in each asset class over time as seen in Figure 26 (on average 52% for fixed income, 19.4% for FX, 15.3% for commodities, 8.4% for equities and 4.9% for energy). However, in risk-terms, this allocation is nowhere close an equal distribution across constituents as shown in Figure 27. The risk allocations per asset class are largely unstable over time and more importantly, there are times at which some asset classes have negative risk contribution.

Figure 26: Volatility-Parity TF Gross Weights



Source: UBS Quantitative Research. The figure presents the sum of gross (absolute) weights for each asset class over time when a volatility-parity weighting scheme is employed (estimated using the past 90 days) for a trend-following strategy. The sample period is from April 1988 to August 2013.

Figure 27: Volatility-Parity TF Risk Allocation



Source: UBS Quantitative Research. The figure presents the percentage contribution to risk for each asset class over time when a volatility-parity weighting scheme is employed (estimated using the past 90 days) for a trend-following strategy. The sample period is from April 1988 to August 2013.

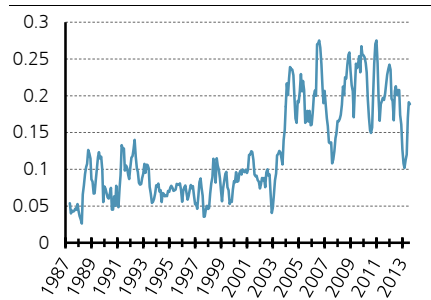
If correlations across the assets were consistently close to zero, then the above plots should follow similar patterns. However, this is definitely not the case. In fact, the implicit outcome from commenting on Figures 18 to 23 in the previous section is that the variance-covariance matrix exhibits some type of segmentation/clustering, in that correlations at the intra-asset-class level tend to be relatively high, but correlations at the inter-asset-class level are relatively low.

Clearly, the zero-correlation assumption that is of the utmost importance for a volatility-parity scheme to be efficient does not appear to hold in practice. In order to support this argument, we next present the rolling average pairwise correlation

Volatility-parity is not equally distributing risk when correlations are non-zero

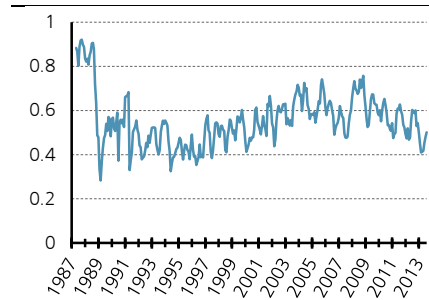
across all futures contracts (Figure 28) and within each asset class (Figures 29 to 33) using a 90-day estimation window. The evidence is clear. Not only is the average pairwise correlation different from zero, but it also fluctuates considerably over time and in fact follows an upward pattern over time for most of the asset classes. More importantly, the average pairwise correlation of the entire universe appears to follow a two-state regime, fluctuating between 0% and 15% in the years 1987-2003 and between 15% and 30% since 2004.

Figure 28: All Contracts



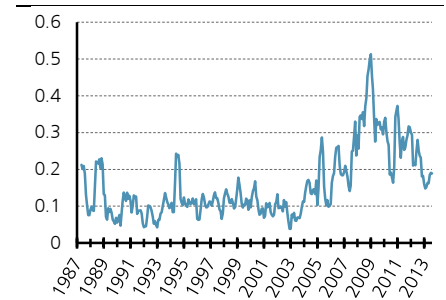
Source: UBS Quantitative Research. Monthly average pairwise correlation across all futures contracts estimated using a 90-day estimation window.

Figure 29: Energy Contracts



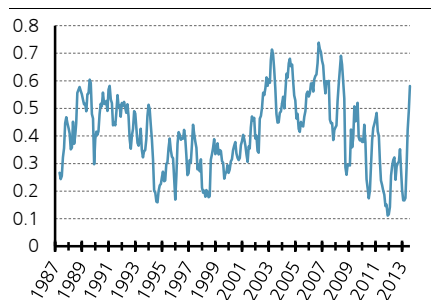
Source: UBS Quantitative Research. Monthly average pairwise correlation across energy contracts estimated using a 90-day estimation window.

Figure 30: Commodity Contracts



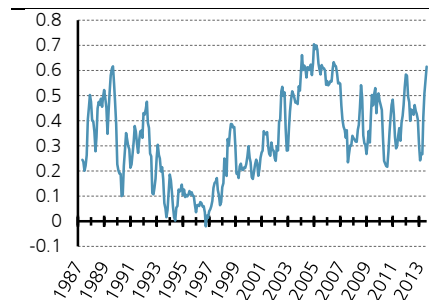
Source: UBS Quantitative Research. Monthly average pairwise correlation across commodity contracts estimated using a 90-day estimation window.

Figure 31: Fixed Income Contracts



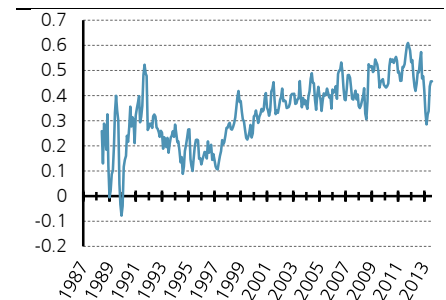
Source: UBS Quantitative Research. Monthly average pairwise correlation across fixed income futures contracts estimated using a 90-day estimation window.

Figure 32: FX Contracts



Source: UBS Quantitative Research. Monthly average pairwise correlation across foreign exchange futures contracts estimated using a 90-day estimation window.

Figure 33: Equity Contracts



Source: UBS Quantitative Research. Monthly average pairwise correlation across equity futures contracts estimated using a 90-day estimation window.

The increasing correlation pattern implies larger degree of co-movement, which in turn, translates into lower diversification benefits. Even the commodity asset class, which has historically been the greater diversifier in the cross-section (Gorton and Rouwenhorst 2006) has exhibited dramatic increase in correlations since 2004 (even though it follows a downward trend towards the end of the sample period). This effect could be largely driven by the so-called financialisation of commodities as discussed in the introduction, following the Commodity Futures Modernization Act (CFMA) in 2000 that allowed investors to hedge commodity risk using futures.

This evidence suggests that volatility-parity is suboptimal as a weighting scheme, as it forcefully suppresses all these correlations to zero and assumes an orthogonal basis formed by all the available assets. In practice, the number of *independent* assets reduces as correlations increase. Assets that exhibit the lowest correlation with the rest of the universe should optimally bear a larger weight than the volatility-parity-implied weight, and similarly, assets that correlate largely with the rest of the universe should have their volatility-parity weight shrunk as they don't add much to the overall diversification.

What has been described in simple (or maybe not-that-simple) terms in the previous paragraph lies at the heart of *true Risk Parity*. We turn our attention to it.

True, but long-only, Risk Parity

Risk Parity (RP) constitutes the extension to a naïve volatility-parity weighting scheme. The objective of this portfolio construction methodology is to distribute the total portfolio risk (volatility) equally across the portfolio constituents or equivalently to equate the weighted contribution to overall portfolio volatility of each and every constituent:

$$w_t^{RP,i} \cdot MCR_t^i = \text{constant}, \quad \forall i \quad (9)$$

where MCR denotes the so-called *marginal contribution to risk* and is defined as the partial derivative of portfolio volatility with respect to the weight of each asset, or in other words the change in portfolio volatility for a small (hence, *marginal*) change in the asset weight:

$$MCR_t^i = \frac{\partial \sigma_t}{\partial w_t^i}, \quad \forall i \quad (10)$$

It can be trivially shown that the sum of weighted marginal contributions to risk is equal to the overall portfolio volatility¹⁸:

$$\sum_{j=1}^{N_t} w_t^j \cdot MCR_t^j = \sigma_t \quad (11)$$

Consequently, the risk-parity objective is to equate all the components of the above summation and effectively set them equal to $\frac{1}{N_t}$ th of the portfolio volatility σ_t .

The solution for the risk-parity objective can be very easily attained using a simple mathematical trick (see Appendix B): maximise the sum of logarithmic weights subject to a risk constraint of target volatility σ_{TGT} :¹⁹

Long-Only Risk-Parity :

Maximise:

$$\sum_{i=1}^{N_t} \log(w_t^i)$$

Subject to:

a) $\sqrt{\mathbf{w}_t^T \cdot \Sigma_t \cdot \mathbf{w}_t} \leq \sigma_{TGT}$

b) $w_t^i > 0, \text{ for every } i$

c) $\sum_{j=1}^{N_t} w_t^j = 1$

(12)

The above formulation has been very popular over the last decade (see e.g. Lee 2011). However, the prescribed recipe is only defined on a long-only framework and cannot be therefore applied to a trend-following strategy (the objective function in the above optimisation is undefined for negative weights).

¹⁸ Contrast this to the inequality: $\sum_{j=1}^{N_t} w_t^j \cdot \sigma_t^j \geq \sigma_t$, where σ_t^j denotes the volatility of asset j at time t .

¹⁹ Strictly speaking, the last constraint $\sum_{j=1}^{N_t} w_t^j = 1$ ("fully-invested" constraint) is not part of the core optimisation process. In practice, the optimised weights are simply rescaled following the optimisation, so that they sum up to 1. See Appendix B for further details.

Objective: equate the risk contributions of portfolio constituents

Long-Short Extended Risk Parity

Thankfully, our recent Global Quantitative Research Monograph *Understanding Risk Parity* (7 February 2013) provides a framework that extends the traditional long-only risk-parity paradigm and allows for long and short positions. This is achieved through:

- introducing asset-specific information in the form of expected returns or some other generic "score" in the optimisation procedure and
- allowing diminishing returns with the size of each position (in other words, the marginal increase in the return drops the higher the position becomes)

Skipping some of the details that are abundantly provided in our previous publication, if μ_t^i denotes the generic "score" for asset i at time t , then the optimisation problem of equation (12) is transformed into:

**Long-Short Extended
Risk-Parity :**

Maximise:

$$\sum_{i=1}^{N_t} |\mu_t^i| \cdot \log(|w_t^i|)$$

Subject to:

- $\sqrt{\mathbf{w}_t^T \cdot \Sigma_t \cdot \mathbf{w}_t} \leq \sigma_{TGT}$
- $w_t^i > 0, \text{ if } \mu_t^i > 0$
- $w_t^i < 0, \text{ if } \mu_t^i < 0$
- $\sum_{j=1}^{N_t} |w_t^j| = 1$

(13)

The key difference between the two approaches is the introduction of the absolute value of the score in the objective function that is also naturally followed by the introduction of the absolute value operator for the portfolio weights in the argument of the logarithm.

More importantly, this new formulation bears a very important feature that is critical for the sections to follow. The sign of the optimised weights, and therefore the type of the positions –long or short– are *entirely determined* by the sign of the scores and *not* by the optimiser. It cannot be stressed enough that:

The sign of the score determines the type (long/short) of position!

- assets with positive scores will have a long position in the portfolio
- assets with negative scores will have a short position in the portfolio

The **Extended Risk-Parity (ERP)** framework, when analytically solved (by trivially extending the solution of the true risk-parity in Appendix B), leads to an extended risk-parity result, in that portfolio constituents do not strictly contribute the same amount of risk to the overall portfolio, but instead, the weighted marginal contribution to risk is *proportional* to the absolute score of each asset:

$$w_t^{ERP,i} \cdot MCR_t^i \propto |\mu_t^i|, \quad \forall i \quad (14)$$

This extended result boils down to the true risk-parity (that however maintains the long-short features), when all portfolio constituents have the same absolute score.

Trend-Following meets Risk-Parity

The extended risk-parity framework can now be very naturally combined with the trend-following strategy. The thinking process is outlined below:

- *Trend-Following*:
 - **Provides** a proper trading rule based on the sign of past return.
 - **Lacks** an efficient risk-weighting scheme when correlations are significantly different from zero.
- *Extended Risk-Parity*:
 - **Provides** a proper risk-weighting scheme that makes use of the entire variance-covariance matrix and distributes the overall risk among the portfolio constituents based on specific rules.
 - **Lacks** a proper scoring methodology of the assets and therefore needs a direct input of the asset scores.

Clearly, what one methodology lacks is offered by the other. For that reason, we decide to use the trend-following signal, the sign of the past 12-month return, as the score of each asset in the risk-parity optimiser:

$$\mu_t^i = \text{sign}(r_{t-12,t}^i) \quad (15)$$

Along these lines the optimisation problem boils down to:

Risk-Parity Trend-Following:

Maximise:

$$\sum_{i=1}^{N_t} \log(|w_t^i|)$$

Subject to:

- a) $\sqrt{\mathbf{w}_t^T \cdot \Sigma_t \cdot \mathbf{w}_t} \leq \sigma_{TGT}$
- b) $w_t^i > 0, \text{ if } \text{sign}(r_{t-12,t}^i) = +1$
- c) $w_t^i < 0, \text{ if } \text{sign}(r_{t-12,t}^i) = -1$
- d) $\sum_{j=1}^{N_t} |w_t^j| = 1$

(16)

It is worth noticing that this choice of score results in a true risk-parity portfolio as equation (14) degenerates into equation (9). Hence, the portfolio volatility is equally split across all portfolio constituents.

Let $w_t^{RP,i}$ denote the net weights that come out of the risk-parity trend-following optimiser. Based on equation (5), we can schematically define the **Risk Parity Trend-Following (RP:TF)** strategy:

$$r_{t,t+1}^{RP:TF} = \frac{\sigma_{TGT}}{\sigma_t^{TF}} \cdot \sum_{i=1}^{N_t} w_t^{RP,i} \cdot r_{t,t+1}^i \quad (17)$$

Before moving to the empirical performance evaluation of this strategy, it is worth highlighting further the key difference between volatility-parity and risk-parity trend-following.²⁰ Given the choice of μ_t^i , equation (14) yields a portfolio weight that is proportional to the reciprocal of the marginal contribution to risk of each asset. The factor of proportionality is in practice absorbed by the fact that absolute weights have to sum up to 1. Along these lines, we can rewrite the risk-parity trend-following weights as follows:

$$w_t^{RP,i} = \frac{(MCR_t^i)^{-1}}{\sum_{i=1}^{N_t} (MCR_t^i)^{-1}} \quad (18)$$

In other words, the risk-parity portfolio is the *inverse-marginal-contribution-to-risk weighted portfolio*, which can be directly contrasted with the *inverse-volatility weighted portfolio*, i.e. the volatility-parity portfolio:

$$w_t^{VP,i} = \frac{(\sigma_t^i)^{-1}}{\sum_{i=1}^{N_t} (\sigma_t^i)^{-1}} \quad (19)$$

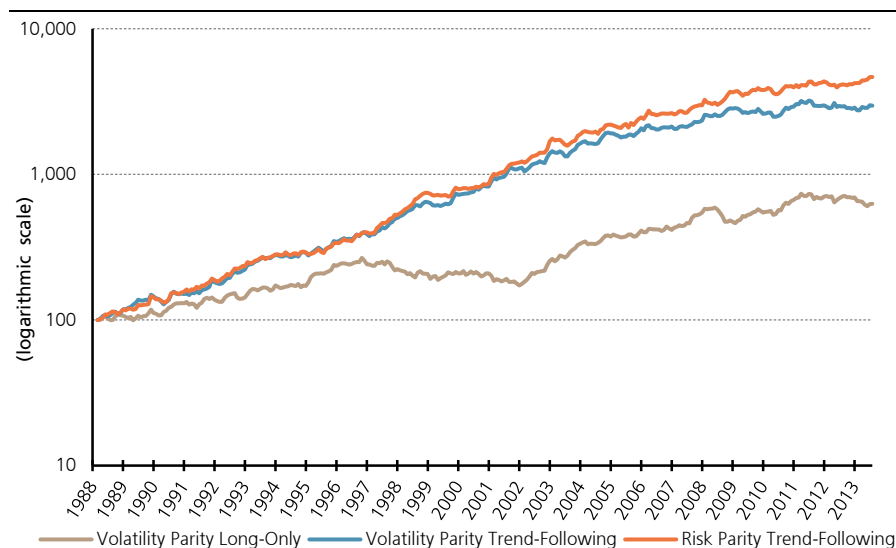
²⁰ For a comparison between volatility-parity and risk-parity schemes on long-only portfolios, see Chaves *et al.* (2011, 2012) and Bhansali *et al.* (2012).

Back to Performance Evaluation

Risk-Parity versus Volatility-Parity

In Figure 34 (identical to Figure 1 but in logarithmic scale), we compare the cumulative returns of the long-only and trend-following volatility-parity strategies with the newly introduced risk-parity trend-following strategy, using a 10% target level of volatility. Over the long-run, the trend-following strategies have a large degree of co-movement, which is expected to a certain degree, given that they employ the same set of long and short positions at the end of month. They only differ in the weighting scheme that they employ.

Figure 34: Cumulative Returns



Source: UBS Quantitative Research. The figure presents the cumulative returns of a volatility-parity long-only strategy, a volatility-parity 12-month trend-following strategy and a risk-parity 12-month trend-following strategy. All risk measures related to the weighting schemes are estimated using the past 90 days. All strategies are targeting a volatility of 10%. The scale is logarithmic. The sample period is from April 1988 to August 2013.

As already mentioned, in a universe of almost zero cross-asset correlations (like the period between 1987 and 2003 as shown in Figure 28), when a covariance matrix is very close to a diagonal matrix (or at least block-diagonal with some non-zero intra-asset-class correlations), volatility-parity and risk-parity solutions are effectively indistinguishable. This is indeed the period that the two trend-following lines in the figure above are very close to each other. However, when correlations take significant non-zero levels (like the most recent period from 2004 onwards), a risk-parity allocation system will underweight in relative terms the assets that are more correlated on average with the universe and accordingly overweight the assets that have low average correlation with the universe and therefore exhibit larger diversification properties. This differentiation in the weight allocation appears to be driving the outperformance of the risk-parity solution in the above chart.

To get better insight of the relative performance and ranking of the strategies, Figure 35 presents various performance statistics as calculated using monthly return series and Figures 36 to 40 visualise some of the statistics (ex-post and rolling Sharpe ratio, Calmar ratio and drawdowns, correlations with benchmark indices). For statistical robustness, Panel D of Figure 35 presents p-values of paired Wilcoxon (1945) signed-rank tests (see Appendix C for further details), in order to assess whether the average returns between any two strategies differ. We present both two-sided and one-sided p-values.

When correlations become significant, risk-parity outperforms volatility-parity

Figure 35: Long-Only, Trend-Following and Risk-Parity

Panel A: Performance Statistics			
	<i>VP: LO</i>	<i>VP: TF</i>	<i>RP: TF</i>
Ann. Geometric Mean (%)	7.49	14.26	16.31
T-statistic (Newey-West)	3.19	6.54	7.71
Ann. Volatility (%)	11.52	10.96	10.83
Skewness	-0.14	0.38	0.57
Kurtosis	3.04	3.30	3.84
Max Drawdown (%)	35.22	14.20	10.65
Sharpe Ratio (annualised)	0.69	1.28	1.46
Sortino Ratio (annualised)	1.11	2.70	3.38
Calmar Ratio	0.21	1.00	1.53

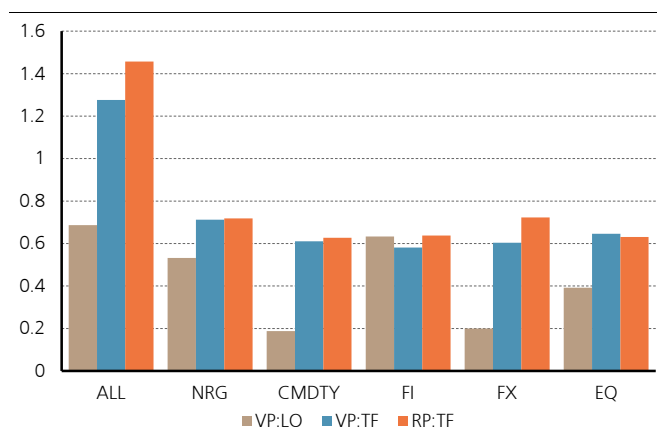
Panel B: Correlation Matrix			
	<i>VP: LO</i>	<i>VP: TF</i>	<i>RP: TF</i>
Volatility-Parity: Long-Only	1.00		
Volatility-Parity: Trend-Following	0.29	1.00	
Risk-Parity: Trend-Following	0.17	0.79	1.00

Panel C: Correlations with Benchmark Indices			
	<i>VP: LO</i>	<i>VP: TF</i>	<i>RP: TF</i>
<i>Commodity Benchmarks:</i>			
- DJ UBS Commodity	0.69	0.10	0.08
- S&P GSCI Commodity	0.58	0.11	0.07
<i>Fixed Income Benchmarks:</i>			
- JPM Global Bond Index	0.69	0.31	0.21
<i>FX Benchmarks:</i>			
- Trade-Weighted USD	-0.54	-0.14	-0.09
- USD/CHF	-0.59	-0.21	-0.11
- USD/JPY	-0.45	-0.18	-0.13
<i>Equity Benchmarks:</i>			
- MSCI World (DM)	0.52	-0.01	0.00
- MSCI Emerging Markets	0.37	-0.05	0.00

Panel D: Two-Sample Paired Tests		
	Two-Sided ($H_0: "="$, $H_1: "\neq"$)	One-Sided ($H_0: "="$, $H_1: ">"$)
<i>VP: TF vs. VP: LO</i>	6.90%	3.45%
<i>RP: TF vs. VP: LO</i>	0.99%	0.50%
<i>RP: TF vs. VP: TF</i>	5.39%	2.70%

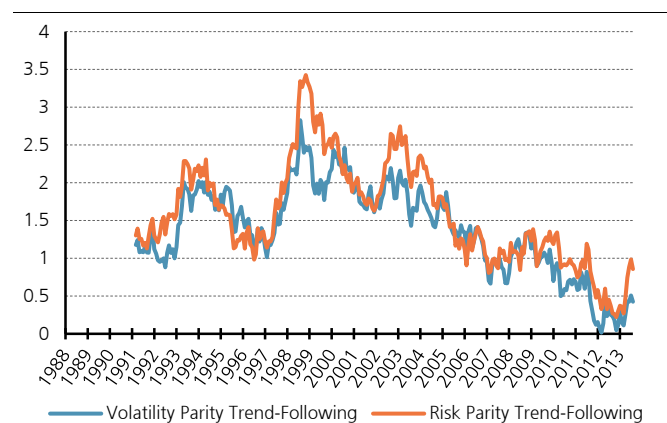
Source: UBS Quantitative Research. The figure reports in Panel A various performance statistics using monthly returns for the volatility-parity long-only strategy (*VP: LO*), the volatility-parity 12-month trend-following strategy (*VP: TF*) and the risk-parity 12-month trend-following strategy (*RP: TF*). The t-statistic of the mean return is calculated using Newey and West (1987) standard errors. The Sortino ratio is defined as the annualised arithmetic mean return over the annualised downside volatility. The Calmar ratio is defined as the annualised geometric mean return over the maximum drawdown. Panel B presents the correlation matrix between the strategies and Panel C presents correlations between the strategies and various benchmarks across all asset classes. All correlation estimates with absolute value larger than 0.50 are presented in red. Panel D presents the p-values from two-sided and one-sided paired Wilcoxon (1945) signed-rank tests for the difference in the monthly average returns between the three strategies. The sample period for all statistics is from April 1988 to August 2013, except for correlations with DJ UBS Commodity index, which is available since February 1991.

Figure 36: Sharpe Ratio Improvement



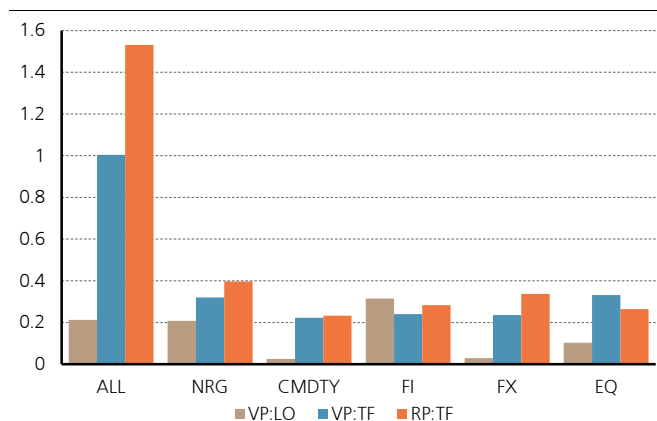
Source: UBS Quantitative Research. The figure presents the annualised Sharpe ratio (calculated using monthly returns) of the volatility-parity long-only strategy (*VP:LO*), the volatility-parity 12-month trend-following strategy (*VP:TF*) and the risk-parity 12-month trend-following strategy (*RP:TF*). It also presents the Sharpe ratio of strategies formed within each asset class. All risk measures related to the weighting schemes are estimated using the past 90 days. All strategies are targeting a volatility of 10%. The sample period is April 1988 - August 2013.

Figure 37: Rolling Sharpe Ratio



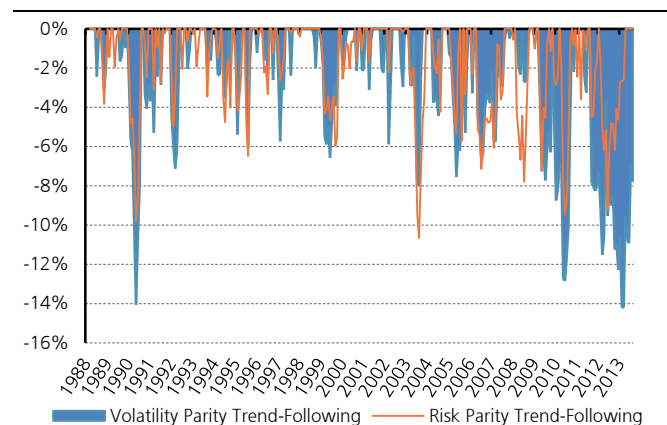
Source: UBS Quantitative Research. The figure presents a 36-month rolling annualised Sharpe ratio calculation (using monthly returns) of the volatility-parity 12-month trend-following strategy and the risk-parity 12-month trend-following strategy. All risk measures related to the weighting schemes are estimated using the past 90 days. The strategies are targeting a volatility of 10%. The sample period is April 1988 - August 2013.

Figure 38: Calmar Ratio Improvement



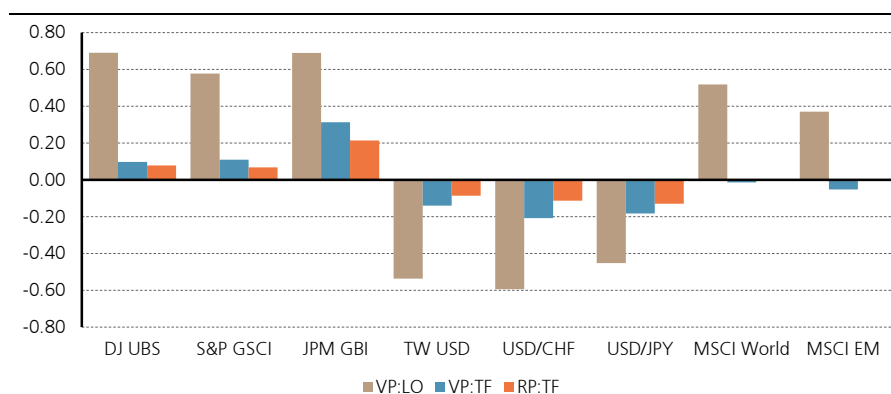
Source: UBS Quantitative Research. The figure presents the Calmar ratio (calculated using monthly returns) of the volatility-parity long-only strategy (*VP:LO*), the volatility-parity 12-month trend-following strategy (*VP:TF*) and the risk-parity 12-month trend-following strategy (*RP:TF*). It also presents the Sharpe ratio of strategies formed within each asset class. All risk measures related to the weighting schemes are estimated using the past 90 days. All strategies are targeting a volatility of 10%. The sample period is April 1988 - August 2013.

Figure 39: Drawdowns



Source: UBS Quantitative Research. The figure presents cumulative drawdowns for the volatility-parity 12-month trend-following strategy and the risk-parity 12-month trend-following strategy. All risk measures related to the weighting schemes are estimated using the past 90 days. The strategies are targeting a volatility of 10%. The sample period is April 1988 - August 2013.

Figure 40: Correlation of Aggregate Strategies with Benchmark Indices



Source: UBS Quantitative Research. The figure presents correlations between the volatility-parity long-only strategy (*VP:LO*), the volatility-parity 12-month trend-following strategy (*VP:TF*) and the risk-parity 12-month trend-following strategy (*RP:TF*) and various benchmark indices across all asset classes. The sample period is from April 1988 to August 2013, except for correlations with DJ UBS Commodity index, which is available since February 1991.

Summarising the main findings:

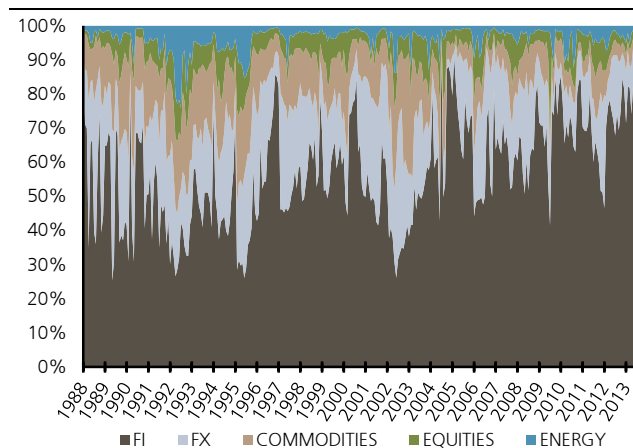
By any measure, risk-parity trend-following dominates

- Risk-parity allocation increases the Sharpe ratio of the volatility-parity trend-following strategy from 1.28 to 1.46. This increase appears to be fairly robust over time, as can be seen in the 36-month rolling Sharpe ratio calculation in Figure 37.
- The hypothesis of equality in the mean return of the strategies is strongly rejected with a two-sided p-value of 5.39% and a one-sided p-value of 2.70% (in favour of the risk-parity trend-following strategy).
- This enhancement is largely driven by significantly reducing the drawdowns of the naïve volatility-parity scheme as seen in Figure 39. Indicatively, the maximum drawdown is reduced by 25% from 14.20% down to 10.65%, which, in turn, is translated into a Calmar ratio of 1.53 for the risk-parity strategy compared to 1.00 for volatility-parity trend-following strategy and a mere 0.21 for the long-only equivalent.
- The risk-parity weighting scheme results in even smaller point estimates of correlation with respect to all benchmark indices that we tested, even though the drop appears to be insignificant. Overall, the large benefit stems from switching from a long-only framework into a long-short trend-following one and not really from the weighting scheme employed.

Risk Allocation

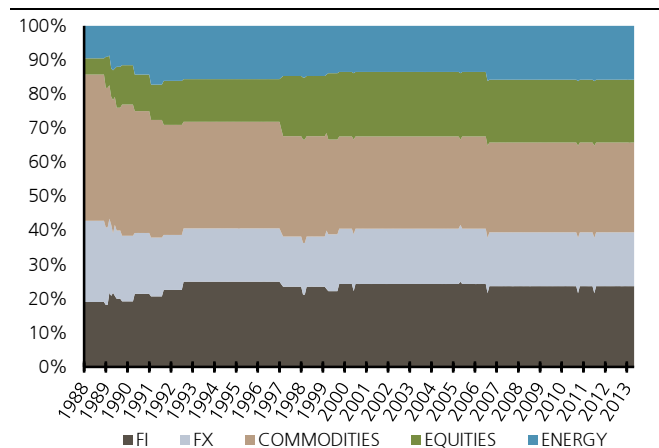
In order to evaluate the efficiency of the risk allocation of the risk-parity methodology and to also compare the portfolio composition with that of volatility-parity, we replicate the analysis presented in Figures 26 and 27; Figure 41 presents the gross weight of each asset class and Figure 42 presents the percentage contribution to risk of each asset class over time.

Figure 41: Risk-Parity Trend-Following Gross Weights



Source: UBS Quantitative Research. The figure presents the sum of gross (absolute) weights for each asset class over time when a true risk-parity weighting scheme is employed (estimated using the past 90 days) for a trend-following strategy. The sample period is from April 1988 to August 2013.

Figure 42: Risk-Parity Trend-Following Risk Allocation



Source: UBS Quantitative Research. The figure presents the percentage contribution to risk for each asset class over time when a true risk-parity weighting scheme is employed (estimated using the past 90 days) for a trend-following strategy. The sample period is from April 1988 to August 2013.

In order to maintain the equal-risk-contribution objective, as is very well demonstrated in the risk allocation figure above, the risk-parity weighting scheme shifts radically between asset classes due to the fast-changing correlation environment. As an example, the average weight in fixed income contracts amounts over time to an average of 57.3%, ranging from as low as 25% to as

high as 92% (contrast this to a rather stable allocation of 52% on average for volatility-parity with a more narrow range between 40% and 65% as presented in Figure 26). However, this rather radically changing allocation is necessary in order to achieve a constant contribution to overall risk per asset class, which is proportional to the number of futures in that class; for fixed income contracts this amounts to 9/38 at the end of the sample period (9 fixed income assets in a universe of 38 futures contracts). The only reason that Figure 42 exhibits (any) risk allocation shifts between asset classes is due to the fact that the overall size of the number of available contracts N_t is not constant over time; nevertheless, at each point in time, each contract contributes $\frac{1}{N_t}$ th of the portfolio volatility.

Revisiting the Recent Underperformance

How much better did risk-parity allocation perform during the recent period of increased correlations and trend-following underperformance? Figure 43 replicates part of Figure 35, but focuses on the very recent sample period between January 2009 and August 2013 that volatility-parity has dramatically underperformed.

Figure 43: Long-Only, Trend-Following and Risk-Parity over 2009-2013

Panel A: Performance Statistics

	<i>VP: LO</i>	<i>VP: TF</i>	<i>RP: TF</i>
Ann. Geometric Mean (%)	5.80	0.88	5.08
T-statistic (Newey-West)	1.30	0.34	1.33
Ann. Volatility (%)	11.86	10.72	9.38
Skewness	-0.37	0.16	-0.07
Kurtosis	3.32	3.10	2.53
Max Drawdown (%)	17.83	14.20	9.48
Sharpe Ratio (annualised)	0.54	0.13	0.58
Sortino Ratio (annualised)	0.82	0.20	0.94
Calmar Ratio	0.33	0.06	0.54

Panel B: Correlations with Benchmark Indices

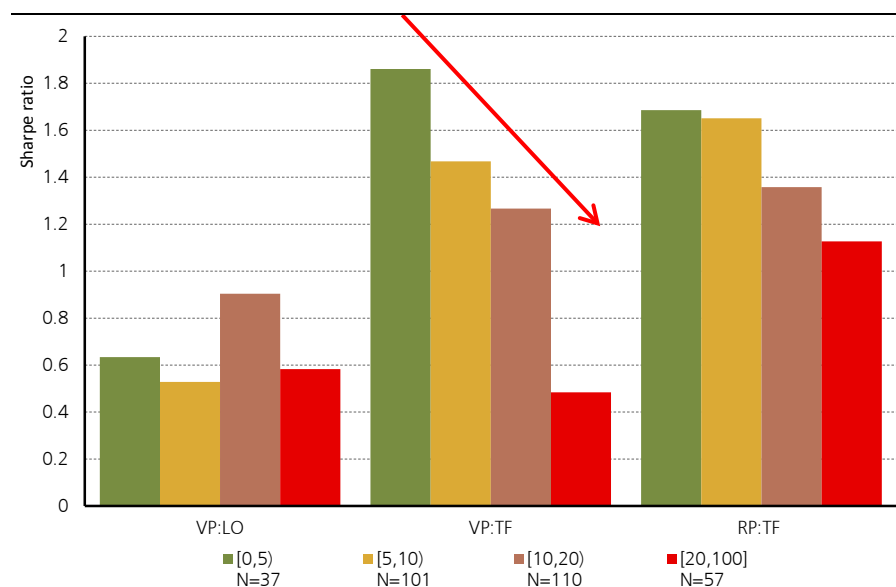
	<i>VP: LO</i>	<i>VP: TF</i>	<i>RP: TF</i>
Commodity Benchmarks:			
- DJ UBS Commodity	0.86	0.27	0.18
- S&P GSCI Commodity	0.80	0.21	0.16
Fixed Income Benchmarks:			
- JPM Global Bond Index	0.73	0.40	0.35
FX Benchmarks:			
- Trade-Weighted USD	-0.56	-0.29	-0.26
- USD/CHF	-0.75	-0.34	-0.33
- USD/JPY	-0.30	-0.26	-0.20
Equity Benchmarks:			
- MSCI World (DM)	0.75	0.21	0.19
- MSCI Emerging Markets	0.73	0.11	0.06

Source: UBS Quantitative Research. The figure reports in Panel A various performance statistics using monthly returns for the volatility-parity long-only strategy (*VP: LO*), the volatility-parity 12-month trend-following strategy (*VP: TF*) and the risk-parity 12-month trend-following strategy (*RP: TF*) for the period between January 2009 and August 2013. Panel B presents correlations between the strategies and various benchmarks across all asset classes. All correlation estimates with absolute value larger than 0.50 are presented in red.

In short, risk-parity revives trend-following over the most recent period and achieves a Sharpe ratio that is marginally better than that of the long-only strategy (0.58 versus 0.54 with the volatility-parity trend-following strategy delivering a mere 0.13).²¹ Furthermore, the performance of the risk-parity trend-following strategy appears much stronger compared to the long-only strategy, if one were to take into account tail events and drawdowns. The equal risk allocation almost halves the maximum drawdown of the long-only strategy. More importantly, the risk-parity weighting scheme is successful in enhancing the performance of the trend-following strategy, while at the same time maintaining the benefit of relatively smaller correlation (hence exposure) to the various benchmark indices.

Finally, augmenting the correlation event study of Figure 25 with the performance of risk-parity trend-following strategy, we document in Figure 44 a less pronounced drop in the performance, with the Sharpe ratio of the strategy ranging between 1.13 and 1.69 across the correlation buckets. Clearly, the more sophisticated risk allocation technique alleviates the performance drops of the volatility-parity trend-following strategy that can be partly attributed to large correlation shifts.

Figure 44: Average Pairwise Correlation Event Study using RP:TF



Source: UBS Quantitative Research. The figure presents the annualised Sharpe ratio of a volatility-parity long-only strategy (*VP:LO*), a volatility-parity trend-following strategy (*VP:TF*) and a risk-parity trend-following strategy (*RP:TF*) for four different states of average pairwise correlation: between 0% and 5%, 5% and 10%, 10% and 20% and above 20%. The number of months *N* for each correlation bucket is shown in the legend. The sample period is from April 1988 to August 2013.

²¹ The two-sided and one-sided p-values of a paired Wilcoxon (1945) signed-rank test on the equality between the mean return of the risk-parity trend-following strategy and the volatility-parity trend-following strategy during the period from January 2009 and August 2013 is 7.60% and 3.80% respectively, showing that the return of the risk-parity strategy is statistically greater than that using the volatility-parity scheme. However, there exists no statistically significant difference between the risk-parity trend-following strategy and the long-only strategy.

Robustness Checks

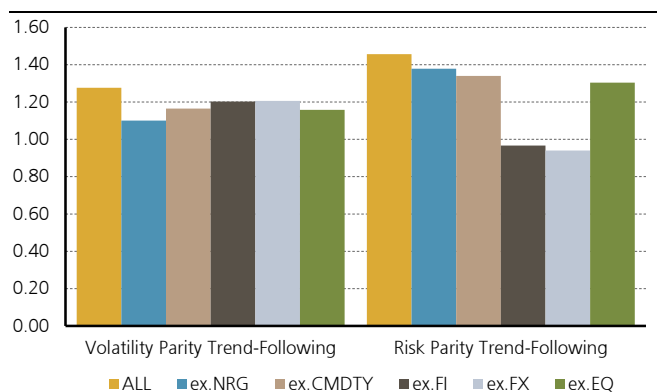
Asset Classes and Day of Implementation

This section looks into the performance of the trend-following strategies in two different scenarios: (a) when a particular asset class is excluded from the portfolio (see Figure 45) and (b) when the momentum signal generation and, consequently, the implementation of the strategy –by default taking place at the end of the month (EOM)– is either delayed by a few days into the following month or "brought forward" before the end of the current month (see Figure 46).

Regarding the first analysis, the evidence shows that volatility-parity trend-following is not dramatically affected by the exclusion of one of the asset classes. However, moving to a risk-parity weighting scheme gives the fixed income and foreign exchange asset classes an important role for the performance of the strategy and exclusion of any of the two reduces the Sharpe ratio of the strategy slightly below 1.00. The reason is that these two asset classes are loosely correlated with the rest of the universe and consequently risk-parity over-weights them in the overall portfolio (in comparison to volatility-parity). The larger weight renders these asset classes important for the overall performance of the strategy; hence, exclusion from a risk-parity portfolio hurts the performance more strongly than for the volatility-parity portfolio.

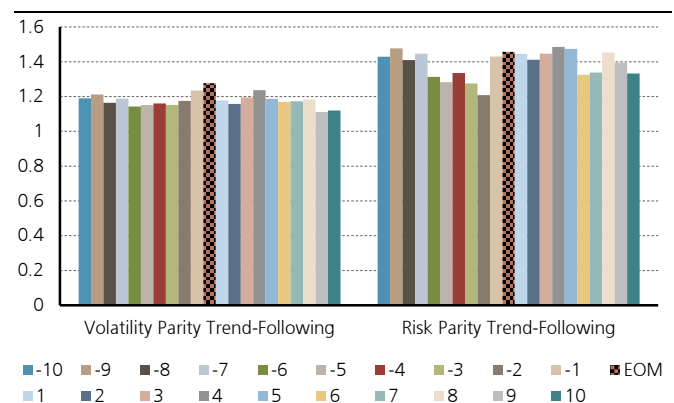
As for the so-called "Day of Implementation" analysis, the results briefly show that there exists no systematic pattern between the profitability of the strategies and the day of signal generation and strategy implementation.

Figure 45: Sharpe Ratio upon Asset Class Exclusion



Source: UBS Quantitative Research. The figure presents the annualised Sharpe ratio calculation (using monthly returns) of the volatility-parity 12-month trend-following strategy and the risk-parity 12-month trend-following strategy when using all asset classes and when excluding one asset class at a time. All risk measures related to the weighting schemes are estimated using the past 90 days. The strategies are targeting a volatility of 10%. The sample period is from April 1988 to August 2013.

Figure 46: The "Day of Implementation" Effects



Source: UBS Quantitative Research. The figure presents the annualised Sharpe ratio calculation (using monthly returns) of the volatility-parity 12-month trend-following strategy and the risk-parity 12-month trend-following strategy when delaying or bringing forward the formation day; default formation day is the end-of-month (EOM). All risk measures related to the weighting schemes are estimated using the past 90 days. The strategies are targeting a volatility of 10%. The sample period is from April 1988 to August 2013.

Extended Risk-Parity and the Choice of "Score"

The risk-parity trend-following strategy that we have tested so far has made use of a "score" characteristic that is simply equal to the momentum signal, the sign of the past 12-month return. This choice renders the solution a true risk-parity allocation as already deduced from the analysis in the previous section.

How dependent the result is to the "score" choice?

However, the general formulation of the long-short extended risk-parity allows for different choices of scores (and not just ± 1), which, in turn, allows for some additional experimentation on the functional form of μ_t^i for each asset, as long as it maintains the same sign as before, so that buy and sell signals are again generated using the past 12-month return. Along these lines, we next test the performance of **Extended Risk-Parity Trend-Following (ERP:TF)** strategies for the following choices of the score:

1. T-statistic of a least-squares linear trend on the past 12-month daily cumulative returns path²² [capped to an absolute value of 2]:

$$\mu_t^i = t.stat(\beta) \text{ from} \quad (20)$$

$$R_\tau^i = \alpha + \beta \cdot \tau + e_\tau, \text{ where } \tau \in [t - 12, t]$$

2. Ratio of past 12-month return over the past volatility²³:

$$\mu_t^i = \frac{r_{t-12,t}^i}{\sigma_t^i} \quad (21)$$

3. Ratio of the sign of past 12-month return over the past volatility:

$$\mu_t^i = \frac{\text{sign}(r_{t-12,t}^i)}{\sigma_t^i} \quad (22)$$

The choice of these scores is of course not exhaustive, but we consider them being relatively representative of the overall potential of the suggested methodology.

The last choice has an implicit tilt towards lower-volatility assets as it over-weights their relative contribution in the objective function of the risk-parity optimisation. In fact, it is worth noticing that this choice is identical to the signed amount of currency units invested in asset i in a simple volatility-parity scheme as per equation (6). It should however be stressed that in the case of equation (22) the risk-parity optimisation does make proper use of the variance-covariance matrix and on top of that involves a low-volatility tilt²⁴.

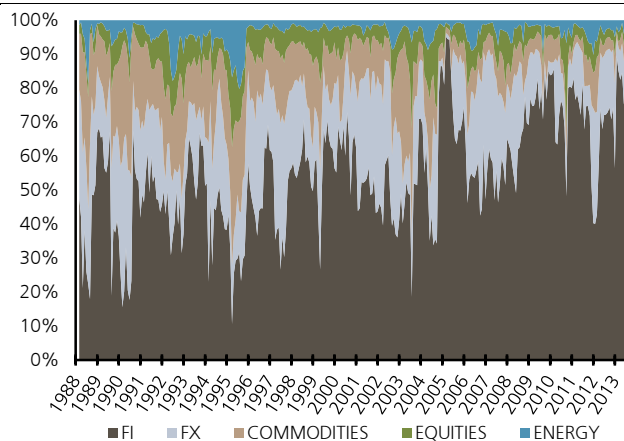
Figures 47 to 52 show the resulting gross weight and risk allocation per asset class.

²² The t-statistic from the linear trend fit is also used as a momentum signal in our Global Quantitative Research Monograph, Momentum and Volatility (24 March 2011) and also proposed in Baltas and Kosowski (2013a).

²³ This choice resembles a Sharpe ratio, but in reality it is not strictly equal to it. The reason is that we make use of a 12-month return in the nominator, so that it ties up with the 12-month signal generation, but we measure volatility in the denominator, using a shorter and more recent estimation window. In this research note we make use of the past 90 days to estimate all these risk measures.

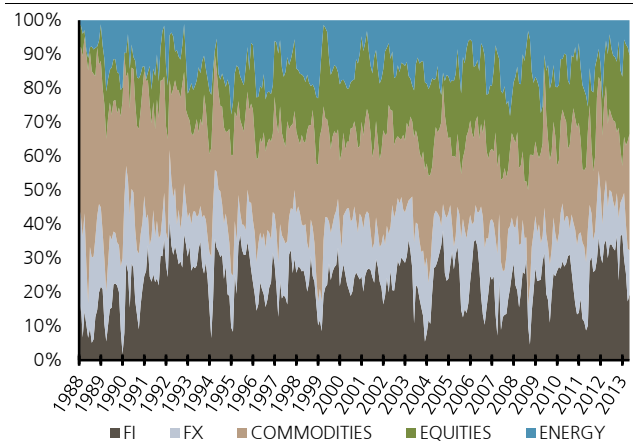
²⁴ Contrast the volatility-parity solution $w_t^{VP,i} = \frac{(\sigma_t^i)^{-1}}{\sum_{i=1}^N (\sigma_t^i)^{-1}}$ to the extended risk-parity solution with a score as of equation (22) (ratio of the sign of past 12-month return over the past volatility) $w_t^{ERP,i} = \frac{(\sigma_t^i \cdot MCR_t^i)^{-1}}{\sum_{i=1}^N (\sigma_t^i \cdot MCR_t^i)^{-1}}$.

Figure 47: $ERP:TF$ [Capped T-Stat] Gross Weights



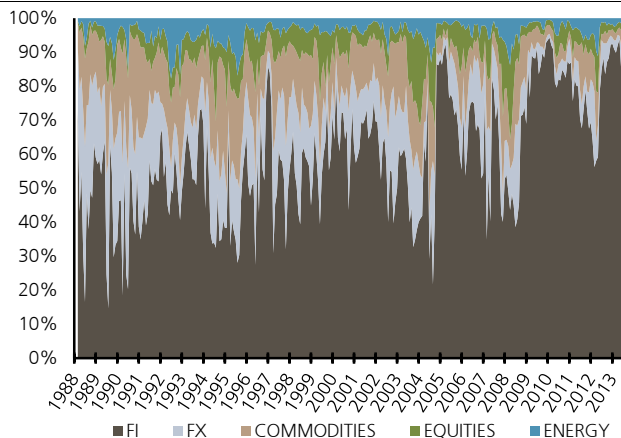
Source: UBS Quantitative Research. The figure presents the sum of gross (absolute) weights for each asset class over time when an extended risk-parity weighting scheme is employed (using the t-statistic of a linear trend fit) for a trend-following strategy. The sample period is from April 1988 to August 2013.

Figure 48: $ERP:TF$ [Capped T-Stat] Risk Allocation



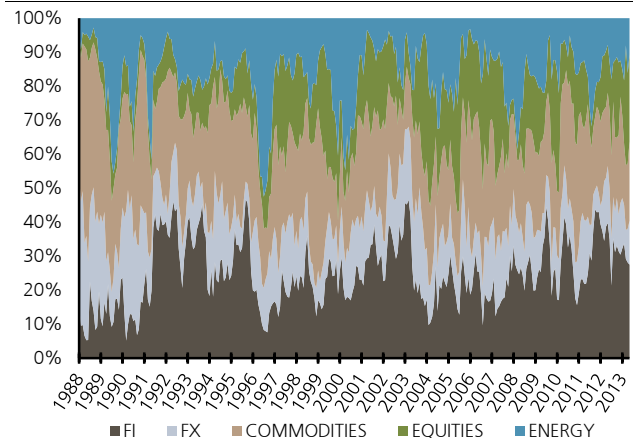
Source: UBS Quantitative Research. The figure presents the percentage constitution to risk for each asset class over time when an extended risk-parity weighting scheme is employed (using the t-statistic of a linear trend fit) for a trend-following strategy. The sample period is from April 1988 to August 2013.

Figure 49: $ERP:TF$ [12M Return/ Volatility] Gross Weights



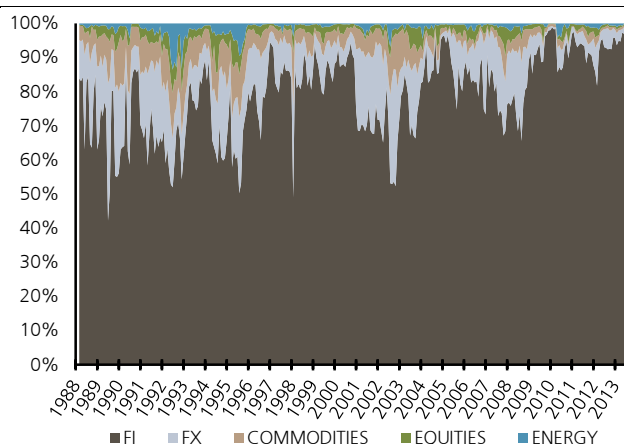
Source: UBS Quantitative Research. The figure presents the sum of gross (absolute) weights for each asset class over time when an extended risk-parity weighting scheme is employed (using the ratio of past 12-month return over volatility) for a trend-following strategy. The sample period is from April 1988 to August 2013.

Figure 50: $ERP:TF$ [12M Return/ Volatility] Risk Allocation



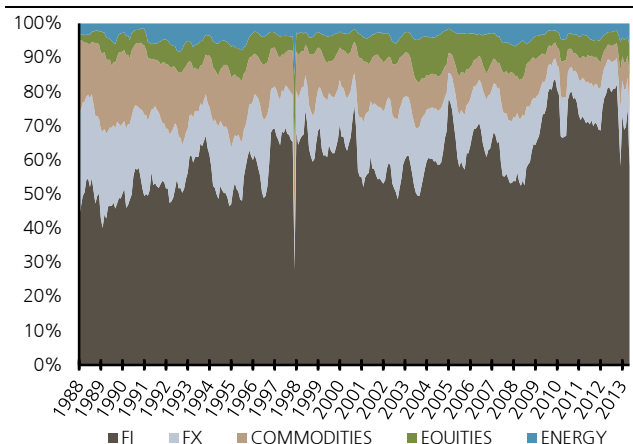
Source: UBS Quantitative Research. The figure presents the percentage constitution to risk for each asset class over time when an extended risk-parity weighting scheme is employed (using the ratio of past 12-month return over volatility) for a trend-following strategy. The sample period is from April 1988 to August 2013.

Figure 51: $ERP:TF$ [Sign(12M Ret.) / Vol.] Gross Weights



Source: UBS Quantitative Research. The figure presents the sum of gross (absolute) weights for each asset class over time when an extended risk-parity weighting scheme is employed (using the ratio of the sign of past 12-month return over volatility) for a trend-following strategy. The sample period is from April 1988 to August 2013.

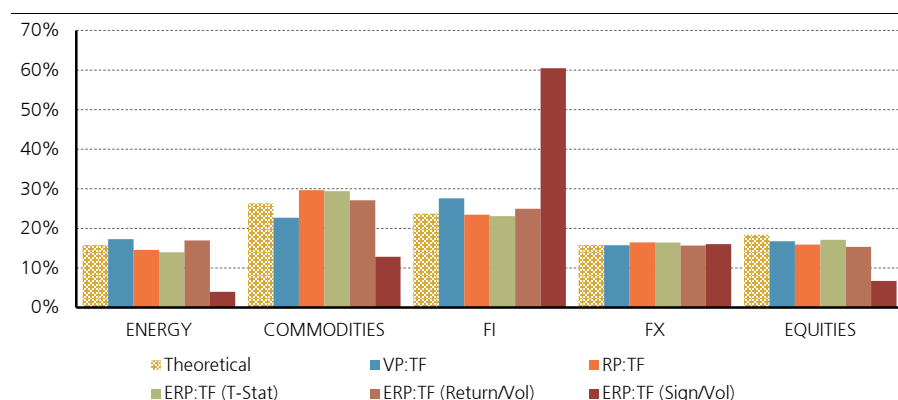
Figure 52: $ERP:TF$ [Sign(12M Ret.) / Vol.] Risk Allocation



Source: UBS Quantitative Research. The figure presents the percentage constitution to risk for each asset class over time when an extended risk-parity weighting scheme is employed (using the ratio of the sign of past 12-month return over volatility) for a trend-following strategy. The sample period is from April 1988 to August 2013.

It is possible to visually identify (using the risk allocation plots) that all extended risk-parity weighting schemes stand between the true risk-parity of Figure 42 (constant allocations over time) and the volatility-parity of Figure 27 (that involves very abrupt risk allocation swings). Fixed income contracts are relatively over-weighted in terms of gross allocation, however, when this is translated into risk allocation, all asset classes share comparable portions of total risk, except when using the last score choice (the normalised-by-volatility sign of past return). As already mentioned, this score choice introduces a very strong low-volatility (and in particular fixed income) tilt. Figure 53 presents the time-series average risk allocation in each asset class for each weighting scheme and verifies the above. For comparability, the figure also involves a theoretical risk-parity allocation estimate by calculating the end-of-sample proportion of contracts of each asset class with respect to the universe (e.g. for fixed income this is 9 out of 38).

Figure 53: Average Risk Allocation per Asset Class and Risk Weighting Scheme



Source: UBS Quantitative Research. The figure presents the average risk allocation per asset class for various risk weighting schemes. The figure also includes a theoretical risk-parity calculation, estimated as the end-of-sample proportion of contracts of each asset class with respect to the universe (38 assets). The sample period is from April 1988 to August 2013.

Turning to performance, Figure 54 reports various statistics over the entire sample period, April 1988 to August 2013 by augmenting parts of Figure 35 with the three *ERP:TF* strategies. We summarise the main findings below:

- Aside from the *ERP:TF* (T-stat) strategy that marginally underperforms the naïve volatility-parity trend-following strategy, all other choices exhibit similar performance. In fact, judging from the two-sample p-values, all risk-parity strategies, but *ERP:TF* (T-stat), offer significantly larger mean return than the volatility-parity trend-following strategy.
- The true risk-parity trend-following strategy, being the only strategy to strictly allocate equal amount of risk to every constituent delivers the most positively skewed distribution of returns with the smallest drawdown and therefore the largest Sortino and Calmar ratios.
- The *ERP:TF* (sign/vol) strategy, having an implicit tilt towards fixed income contracts as explained above achieves the largest Sharpe ratio in the cross-section of strategies (1.51 versus 1.46 for the true risk-parity).
- This outperformance due to the fixed-income tilt makes the risk-parity allocation more immune to changes in average pairwise correlation compared to the naïve volatility-parity counterparty. Figure 55 reproduces one last time the correlation event study using the *ERP:TF* (sign/vol) strategy instead of the true risk-parity of Figure 44 and fails to document a systematic pattern between performance and correlation regime.

Figure 54: Performance Statistics of Extended Risk-Parity Strategies

Panel A: Performance Statistics

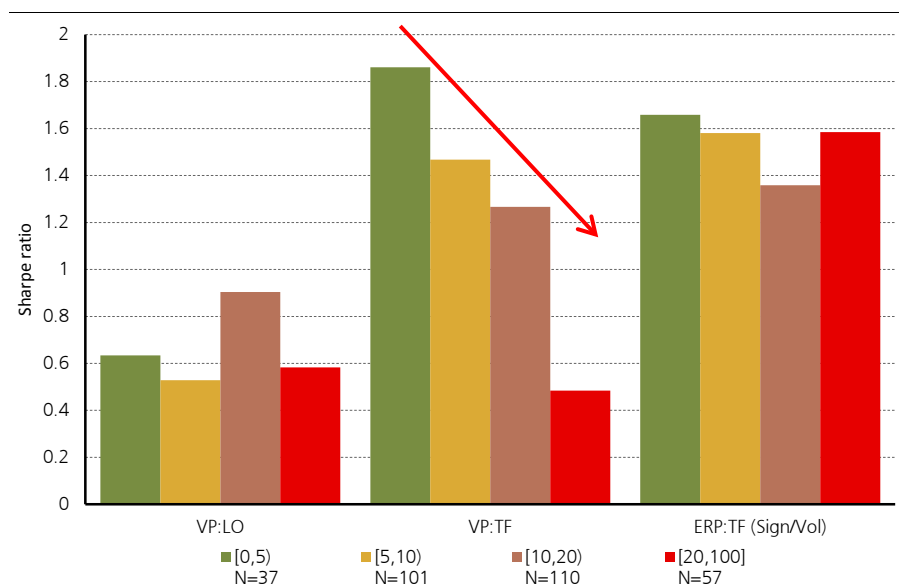
	<i>VP:LO</i>	<i>VP:TF</i>	<i>RP:TF</i>	Extended Risk-Parity		
				T-stat	Return/ Vol	Sign/ Vol
Ann. Geometric Mean (%)	7.49	14.26	16.31	12.91	16.82	17.14
T-statistic (Newey-West)	3.19	6.54	7.71	5.73	9.30	6.80
Ann. Volatility (%)	11.52	10.96	10.83	10.48	11.17	10.94
Skewness	-0.14	0.38	0.57	0.43	0.38	0.26
Kurtosis	3.04	3.30	3.84	3.12	4.13	3.15
Max Drawdown (%)	35.22	14.20	10.65	15.82	11.48	12.84
Sharpe Ratio (annualised)	0.69	1.28	1.46	1.22	1.45	1.51
Sortino Ratio (annualised)	1.11	2.70	3.38	2.59	3.14	3.32
Calmar Ratio	0.21	1.00	1.53	0.82	1.46	1.33

Panel B: Two-Sample Paired Tests against *VP:LO*

	Two-Sided ($H_0: "="$, $H_1: "\neq"$)	One-Sided ($H_0: "="$, $H_1: ">"$)
<i>RP:TF</i>	5.39%	2.70%
<i>ERP:TF</i> (T-stat)	54.24%	72.90%
<i>ERP:TF</i> (Return/Vol)	9.57%	4.79%
<i>ERP:TF</i> (Sign/Vol)	7.56%	3.78%

Source: UBS Quantitative Research. The figure reports in Panel A various performance statistics using monthly returns for the volatility-parity long-only strategy (*VP:LO*), the volatility-parity 12-month trend-following strategy (*VP:TF*), the risk-parity 12-month trend-following strategy (*RP:TF*) and three extended risk-parity trend-following strategies that make use of different score functions. The smallest maximum drawdown and the largest performance ratios in the last four rows are presented with shaded background. Panel B presents the p-values from two-sided and one-sided paired Wilcoxon (1945) signed-rank tests for the difference in the monthly average returns between the various risk-parity strategies and the volatility-parity trend-following strategy. The sample period is from April 1988 to August 2013.

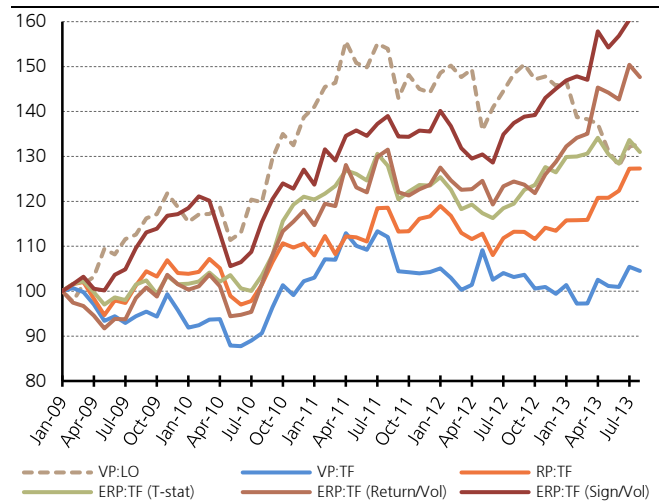
Figure 55: Average Pairwise Correlation Event Study using *ERP:TF*



Source: UBS Quantitative Research. The figure presents the annualised Sharpe ratio of a volatility-parity long-only strategy (*VP:LO*), a volatility-parity trend-following strategy (*VP:TF*) and an extended risk-parity trend-following strategy (*ERP:TF*) that uses the sign of past 12-month return over the past volatility as the score for four different states of average pairwise correlation: between 0% and 5%, 5% and 10%, 10% and 20% and above 20%. The number of months N for each correlation bucket is shown in the legend. The sample period is from April 1988 to August 2013.

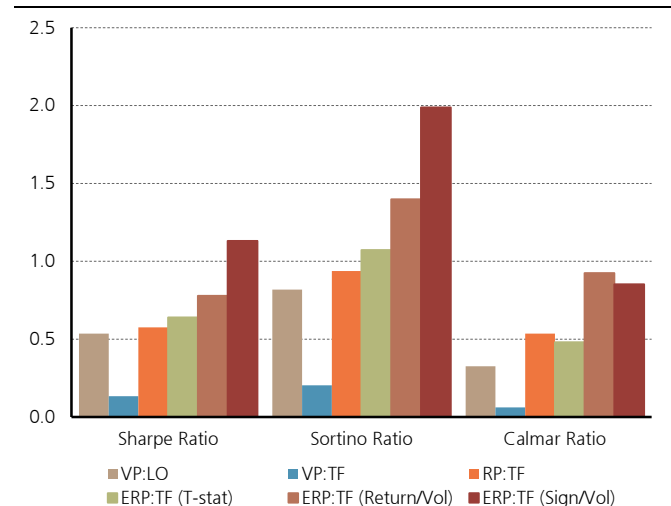
As a final check of risk-parity performance, Figure 56 presents the cumulative returns of the various risk weighted strategies suggested in this note over the most recent period of naïve volatility-parity trend-following underperformance, January 2009 and August 2013. Similarly, Figure 57 presents the respective performance ratios (Sharpe ratio, Sortino ratio, Calmar ratio).

Figure 56: Cumulative Returns 2009-2013



Source: UBS Quantitative Research. The figure presents the cumulative returns of the volatility-parity long-only strategy (*VP:LO*), the volatility-parity 12-month trend-following strategy (*VP:TF*), the risk-parity 12-month trend-following strategy (*RP:TF*) and three extended risk-parity trend-following strategies (*ERP:TF*) that make use of different score functions. The sample period is from January 2009 to August 2013.

Figure 57: Performance Ratios 2009-2013



Source: UBS Quantitative Research. The figure reports the Sharpe ratio, the Sortino ratio and the Calmar ratio (calculated using monthly returns) of the volatility-parity long-only strategy (*VP:LO*), the volatility-parity 12-month trend-following strategy (*VP:TF*) and the risk-parity 12-month trend-following strategy (*RP:TF*) for the period between January 2009 and August 2013.

Irrespective of the score choice, risk-parity weighting schemes when applied to a trend-following strategy deliver better risk-adjusted returns than a volatility-parity long-only strategy. The outperformance is statistically strong, economically significant and robust to various alterations applied in the methodology.

Risk-parity dominates

What about Turnover?

The last part of the analysis of this research paper looks at the turnover estimates of the various strategies that have been constructed.

Figure 58 reports monthly turnover estimates for (i) unlevered strategies, (ii) levered (constant-volatility) strategies when the rebalancing takes place at the end of the month and (iii) levered strategies when the rebalancing takes place at the end of each day. For clarification, in the latter case, momentum trading signals are still generated at the end of each month; however, the portfolio is rebalanced on a daily basis in order to achieve constant volatility. This is the type of strategies that have been tested so far in the previous sections of this research paper.

The benefit of using daily-rebalanced strategies is that we can track the target volatility of the strategy more closely, but this naturally involves more trading. Performance-wise, the switch between daily to monthly rebalancing has insignificant ex-post effects; the Sharpe ratio of volatility-parity long-only strategy drops from 0.69 to 0.68, that of volatility-parity trend-following strategy remains unchanged at 1.28 and finally that of risk-parity trend-following strategy drops from 1.46 to 1.44 for the entire sample period (April 1988 – August 2013).

Figure 58: Monthly Two-way Turnover

	Unlevered	Levered, Monthly Rebalance	Levered, Daily Rebalance
Volatility-Parity Long-Only	6.8%	11.1%	35.1%
Volatility-Parity Trend-Following	25.2%	31.1%	54.6%
Risk-Parity Trend-Following	52.6%	56.3%	82.9%
Extended Risk-Parity Trend-Following:			
- <i>Slope T-statistic of 12M Price Path (capped)</i>	43.9%	48.5%	74.9%
- <i>12M Return / 90-day Volatility</i>	44.0%	49.2%	75.5%
- <i>Sign (12M Return) / 90-day Volatility</i>	38.2%	45.1%	67.1%

Source: UBS Quantitative Research. This figure reports two-way turnover estimates for the volatility-parity long-only strategy, the volatility-parity 12-month trend-following strategy, the risk-parity 12-month trend-following strategy and three extended risk-parity trend-following strategies that make use of different score functions. All strategies are implemented with an unlevered profile, a levered (constant-volatility) profile with monthly rebalancing and a levered (constant-volatility) profile with daily rebalancing. The sample period is from April 1988 to August 2013.

The main findings of this turnover analysis are summarised below:

- Switching from a long-only to a long-short framework implies a relative increase in the trading activity approximately between 50% (levered with daily rebalancing) to even 370% (unlevered).
- Switching from a volatility-parity scheme to a true risk-parity scheme for a trend-following strategy implies a relative increase in the trading activity between 50% (levered with daily rebalancing) and 100% (unlevered).
- For a long-only strategy the trading activity almost doubles when running a constant-volatility strategy with monthly rebalancing in comparison to an unlevered one.
- In contrast, for a trend-following strategy, the additional cost of running a constant-volatility strategy with monthly rebalancing is insignificant. The large majority of trading comes from the shifts between long and short positions.
- Costs become significant, when applying a daily rebalancing scheme, due to the effort to maintain a constant-volatility profile for a trend-following strategy, which increases the trading activity by about 50%-70% compared to a monthly rebalancing scheme.
- Out of all the risk-parity weighting schemes tested, the *ERP:TF* (sign/vol) strategy is the cheapest to rebalance, mainly due to the fixed-income over-weight (which translates into less pronounced weight shifts), while the most expensive is the true risk-parity strategy.

Conclusion

Trend-following strategies have been very profitable historically and have constituted great diversification tools against market downturns like recently during the financial crisis of 2008.

Their main source of profitability is related to the diversification benefit that stems from combining futures contracts across different assets classes, which have exhibited historically very low, if not insignificant, cross-correlations. A simple volatility-parity (inverse-volatility) weighting scheme has been considered appropriate in an effort to bring together assets from different asset classes with very diverse risk profiles. Such a weighting scheme would effectively distribute risk equally among constituents, should their correlations were equal to zero.

Following the impressive performance of 2008, trend-following strategies have performed poorly over the most recent period (2009-2013). This could be either due to momentum patterns not being strong and significant anymore and/or due to the weighting scheme not meeting its primary objective, that to spread the risk evenly between constituents. This research note addresses the latter effect. One potential reason for this underperformance could be the increased co-movement across assets and across asset classes that has been documented over the last decade.

Using a recent extension of the risk-parity paradigm to a long-short framework, we construct trend-following strategies with appropriately optimised weights so that each constituent contributes strictly the same amount of risk to the overall portfolio, after taking into account all the correlations amongst them.

Not only does the suggested methodology result in a portfolio that achieves significantly larger return than its volatility-parity counterparty over the entire sample period that we test (April 1988 – August 2013), but more importantly it manages to revive the trend-following strategy over the very recent period of significant underperformance (January 2009 – August 2013) by rendering it relatively more immune to sharp correlation increases.

Irrespective of the above enhancement, it is true that trend-following patterns have been significantly reduced over the recent period and the "why" question remains open. However, we do consider our approach as a genuine improvement to the construction of portfolios across asset-classes as it aims to take advantage of any available diversification within the investable universe.

APPENDIX

A. Working with Futures Contracts

A.1 Constructing Continuous Price Series

Constructing continuous futures price-series is not straightforward, because futures contracts are short-lived instruments that expire from time to time. The standard approach is to splice together the time-series of neighbouring contracts effectively mimicking an investor that trades on a particular contract and then, prior to expiration, rolls-over the position onto a different contract of the same underlying entity that matures at a later date.

In applying this scheme it is reasonable to use the most heavily traded contract (by open interest or volume) at each point in time, which is almost always the *closest-to-mature* contract (also known as the *front* contract). The roll-over should then naturally take place when liquidity jumps to the *next-to-mature* contract (also known as the *first-back* contract). This way, we effectively make use of the most liquid contract at each point in time. This approach is fairly standard in futures-based literature (e.g. see de Roon, Nijman and Veld 2000, Moskowitz *et al.* 2012, Baltas and Kosowski 2013a, 2013b).

Even though the suggested approach seems reasonable, it gives rise to an important issue that has to be appropriately taken care of. When a roll-over is instructed due to a liquidity shift, there is no particular reason why the futures price of the two neighbouring contracts that participate in a roll-over should be the same. Hence, splicing the price series of these contracts together would give rise to a price jump, which, in turn, would give rise to a fictitious return that is not capitalised by the investor. Even worse, if the futures prices on a particular underlying entity are almost always in contango or backwardation, the above effect would systematically bias the average return of the asset upwards or downwards respectively.

Clearly, special care needs to be taken to alleviate this issue. Let F_{t,T_1} and F_{t,T_2} denote the time t futures prices of two contracts of the same underlying with maturities T_1 and T_2 respectively. In order to remove the price jump of size $|F_{t,T_2} - F_{t,T_1}|$ from the continuous price series if a roll-over is to take place at time t , we can:

- add **backwards** to the entire price path up to time t the price **difference** $F_{t,T_2} - F_{t,T_1}$ (which, if negative, would result in subtraction), or
- multiply **backwards** the entire price path up to time t with the price **ratio** $F_{t,T_2}/F_{t,T_1}$.

Whichever the chosen scheme (trivially called the "*backwards-difference*" and "*backwards-ratio*" adjustments), the entire price path leading to a roll-over will be appropriately shifted so that the prices of the neighbouring contracts are the same and equal to the price of the more recent contract. However, there exists one last complication. The backwards-difference adjustment distorts historical percentage changes (e.g. the return between 100 and 105 is not the same as the return between $100 + k$ and $105 + k$) and is therefore inappropriate for use in a back-testing environment. In fact, if the futures prices on a particular underlying entity are almost always in contango or backwardation, the backwards-difference adjustment would systematically bias the average return downwards or upwards

respectively. A second disadvantage is that the backwards-difference adjustment can result in negative historical prices for markets that are in strong backwardation.

Both these issues are resolved if one uses the backwards-ratio adjustment (the return between 100 and 105 is the same as the return between 100k and 105k). For that reason, we decide to use the backwards-ratio adjustment methodology for the roll-over of the contracts, which results in tradable price series to be used for back-testing purposes. For the sake of completeness, the above schemes can be applied on a forward basis by always shifting the "future" of the futures prices after a roll-over. The disadvantage of the forward adjustment, which however does not affect our purpose, is that it results in the most recent futures prices being very different to the prevailing ones due to the consecutive forward adjustments; a backwards scheme would have recent prices that match the prevailing prices in the market and any significant deviation would only become pronounced in the far past prices.

In order to obtain backwards-ratio adjusted continuous price series with the roll-overs taking place when liquidity/trading activity shifts between contracts, we make use of Bloomberg's *generic* contracts that track the active contract. The adjustment scheme can be set in the <GFUT> screen of Bloomberg.

Further information on the roll-over schemes and adjustments can be found in Carchano and Pardo (2009) and Masteika, Saulius and Alexander Janes (2012).

A.2 Calculating Holding Period Returns

Having constructed continuous futures price-series, the next challenge is to calculate holding period returns in excess of the prevailing risk-free rate. The rationale of the suggested approach is outlined in Baltas and Kosowski (2013a).

In theory, futures contracts are unfunded investments and no initial capital is required to enter a new position. In practice, opening a new position requires posting a cash collateral known as the initial margin. In order to estimate holding period returns, let F_t and F_{t+1} denote the (continuous) futures price at the end of month t and $t + 1$ respectively. Entering a futures position at time t would require an initial margin M_t that earns the prevailing risk-free rate, r_t^f , hence at the end of the month it is expected to grow to $M_t(1 + r_t^f)$. Assuming no variation margin payments, the cash amount in the margin account at the end of the month residing would be equal to $M_t(1 + r_t^f) + (F_{t+1} - F_t)$. Based on that, the holding period return over the course of the month in excess of the risk-free rate is:

$$r_{t,t+1}^{xs} = \frac{[M_t(1 + r_t^f) + (F_{t+1} - F_t)] - M_t}{M_t} - r_t^f = \frac{F_{t+1} - F_t}{M_t} \quad (23)$$

This result strongly related to the so-called *Return on Margin* quantity. Instead of making assumptions for initial margin requirements, we can take the extreme case scenario of a fully collateralised position, i.e. the initial margin is assumed to be equal to the prevailing futures price at time t . If anything, this assumption subsumes the no-variation-margin assumption that was made above. Following that, the fully-collateralised monthly return on a futures contract in excess of the risk-free rate is given by the following equation (which is identical to equation (1) of this research paper):

$$r_{t,t+1} \equiv r_{t,t+1}^{xs,fc} = \frac{F_{t+1} - F_t}{F_t} \quad (24)$$

Interestingly, the functional form resembles a total return calculation of a cash equity transaction, but it should be stressed that this is now an excess return on top of the assumed risk-free rate.

B. Solving for Risk Parity

Risk parity methodology solves for portfolio weights, so that every asset is contributing the same amount of risk to the overall portfolio, i.e.

$$w_t^{RP,i} \cdot MCR_t^i = \text{constant}, \quad \forall i \quad (25)$$

Our recent Quant Keys *Risk parity with different risk measures* (10 July 2013) offers a very simple mathematical trick, in order to solve for the above objective. That involves re-phrasing the problem into a non-linear constrained optimisation problem:

- Maximise the sum of logarithmic weights $\sum_{i=1}^{N_t} \log(w_t^i)$
- Subject to a risk constraint of target volatility $\sigma_{w,t} \equiv \sqrt{w_t^T \cdot \Sigma_t \cdot w_t} \leq \sigma_{TGT}$

The additional "fully-invested" constraint $\sum_{j=1}^{N_t} w_t^j = 1$ can be applied at a later stage so to facilitate the optimisation procedure.

We first form the Lagrangian:

$$L(w_t) = \sum_{i=1}^{N_t} \log(w_t^i) - \lambda \cdot (\sigma_{w,t} - \sigma_{TGT}) \quad (26)$$

We then calculate all partial derivatives with respect to each weight w_i :

$$\frac{\partial L(w_t)}{\partial w_t^i} = \frac{1}{w_t^i} - \lambda \cdot \underbrace{\frac{\partial \sigma_{w,t}}{\partial w_t^i}}_{\equiv MCR_t^i}, \quad \forall i \quad (27)$$

Setting the above expression equal to zero leads to equation (25), with the constant being the reciprocal of the Lagrangian multiplier λ .

As a final step, all weights are rescaled by their sum, so that they all sum up to 1. The fact that volatility exhibits positive homogeneity (for a scaling constant κ , $\sigma_{\kappa \cdot w} = \kappa \cdot \sigma_w$) renders the *MCR* scale-invariant as is easily deduced by:

$$\frac{\partial \sigma_{\kappa \cdot w_t}}{\partial (\kappa w_t^i)} = \frac{\partial \sigma_{w_t}}{\partial w_t^i} = MCR_t^i, \quad \forall i \quad (28)$$

This means that the rescaled weights satisfy the risk-parity objective of equation (25), as the normalisation constant (the sum of unadjusted weights) is absorbed by the constant of equation (25). Delaying the application of the "fully-invested" constraint until after the optimisation helps computationally and does not alter the end result in terms of the risk-parity objective.

C. Two-sample "Difference in Mean" Tests

C.1 Overview of Tests

When we want to compare two sample series realisations of two random variables X and Y –in our case two return series– in order to statistically infer whether their respective means are statistically different, we can employ two-sample tests. This section gives a broad overview of these tests. There exist various versions of these tests, depending on whether the two samples (i) follow a normal distribution or not, (ii) have the same or unequal variance, (iii) have the same or unequal sample size and (iv) are independent to each other or "*paired*". Depending on the answer to the above four criteria, different versions of tests should be employed.

Which test to use? 4 criteria

Without getting into many details, if the two samples follow a **normal distribution**, the test takes the general form of the ratio between the difference in the means of the two samples and *some* measure of standard error of the pair:

Samples with Normal distribution

$$t_{X,Y} = \frac{\bar{X} - \bar{Y}}{se(X,Y)} \quad (29)$$

The denominator in the above ratio depends on the answer to criteria (ii) to (iv):

- Independent samples, equal variance (independent of the sample sizes): the test is known as the **Student's t-test** and the denominator is the *pooled standard error*:

$$se(X,Y) = \sqrt{\left(\frac{1}{n_X} + \frac{1}{n_Y}\right) \frac{(n_X - 1)s_X^2 + (n_Y - 1)s_Y^2}{n_X + n_Y - 2}} \quad (30)$$

with s_X and s_Y being the sample variances of the two samples with sizes n_X and n_Y respectively.

- Independent samples, unequal variance (independent of the sample sizes): the test is known as the **Welch's t-test** (Welch 1947) and the denominator is *not* the pooled standard error:

$$se(X,Y) = \sqrt{\frac{s_X^2}{n_X} + \frac{s_Y^2}{n_Y}} \quad (31)$$

- Paired samples (obviously with the same size n and variance): the test is known as the **difference paired test** and the denominator is the standard deviation of the difference in the means (which in this case is equal to the mean of the difference of all $X - Y$ pairs due to the sample size equality):

$$se(X,Y) = \sqrt{\frac{var(X - Y)}{n}} = \sqrt{\frac{1}{n}(s_X^2 + s_Y^2 - 2s_X s_Y \rho_{X,Y})} \quad (32)$$

If the correlation $\rho_{X,Y}$ is largely positive (as it tends to be for paired samples), then the standard error is significantly reduced relative to the same-size equivalent for independent samples as of equation (30), i.e. $\sqrt{\frac{1}{n}(s_X^2 + s_Y^2)}$. This results in larger statistical power compared to the standard Student's t-test.

When the two samples of X and Y are not normally distributed, hence exhibit non-zero skewness and/or excess kurtosis, the above tests are not appropriate. Instead, alternative **non-parametric tests** should be used. The choice of test depends again on whether the samples are independent or paired:

- Independent samples (independent of the sample sizes): the test is known as the **Wilcoxon Rank-Sum test** (Wilcoxon 1945) or the **Mann-Whitney U test** (Mann and Whitney 1947) and calculates a test statistic as the sum of the ranks of one of the two samples when all realisations of X and Y ($n_X + n_Y$ in total) are jointly ranked. The test statistic follows a prescribed distribution (tabulated for small samples, close to normal for large samples) and therefore p-values can be calculated.
- Paired samples (obviously with the same size n): the test is known as the **Wilcoxon Signed-Rank test** (Wilcoxon 1945) and calculates the absolute value of the sum of all signed ranks of the differences of the $X - Y$ pairs ($2n$ in total). The test statistic follows a prescribed distribution (tabulated for small samples, close to normal for large samples) and p-values can be calculated for the null hypothesis that the median difference in the pairs is zero.

C.2 Which test do we need for our purposes?

Figures 35 and 54 of this research paper perform statistical tests of equality in the mean return between long-only and various trend-following strategies. In order to identify the appropriate test to use, we need to address a number of questions:

1. Normality or non-Normality? Skewness and kurtosis estimates of the return series of the various strategies from Figures 35 and 54 show that probably with the exception of the long-only strategy (skewness of -0.14, kurtosis of 3.04) all other trend-following strategies could exhibit non-normality (skewness in the range 0.26 to 0.57, kurtosis in the range 3.12 to 4.13). To test for normality Figure 59 reports p-values of Shapiro and Wilk (1965) normality tests in the monthly return series of all strategies. As expected, the trend-following strategies (excluding the one using the sign/volatility score) strongly reject the null of a normal distribution at the 5% or 1% level. This evidence clearly suggests the use of a non-parametric two-sample test.
2. Equality of Variances? Even though this is irrelevant to a non-parametric test we can simply argue that all strategies are implemented with a constant volatility of 10% so they are expected to have similar variances. Performing simple tests for equality of variances (like the Bartlett test) between any of pair of strategies yields p-values that range roughly between 26% and 86%, which means that the equality of variances appears to empirically hold.
3. Equality of Size? This is fairly straightforward. All strategies have return series between April 1988 and August 2013, i.e. 308 months.
4. Independent or Paired Samples? This is probably the most important question. It is obvious that all pairs of strategies are indeed paired. At the end of each month, trend-following signals are generated and the only difference between the strategies is the employed weighting scheme. Along these lines, each month's return of one strategy is by construction uniquely paired with the same month's return of another strategy.

Following the above, the test choice is obvious. We need to make use of a non-parametric test that is designed for paired samples; this is the Wilcoxon Signed-Rank test that is easily run within the R statistical package using the command: `wilcox.test(X,Y,paired=TRUE)`.

When the normality assumption does not hold

Figure 59: Normality Tests

Strategy	p-value
VP: LO	76.44%
VP: TF	4.13%
RP: TF	0.06%
ERP: TF (T-stat)	0.43%
ERP: TF (Return/Vol)	0.53%
ERP: TF (Sign/Vol)	20.79%

Source: UBS Quantitative Research. The figure documents the p-values from Shapiro and Wilk (1965) normality tests for a range of strategies using monthly returns. The sample period is from April 1988 to August 2013.

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